The Einstein-Jordan conundrum and its relation to ongoing foundational research in local quantum physics

Dedicated to the memory of Claudio D'Antoni

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Abstract

We demonstrate the extraordinary modernity of the 1924/25 "Einstein-Jordan fluctuation conundrum", a Gedankenexperiment which led Jordan to his quantization of waves published as a separate section in the famous Born-Heisenberg-Jordan 1926 "Dreimännerarbeit". The thermal nature of energy fluctuations caused by the restriction of the QFT vacuum to a subvolume remained unnoticed mainly because it is not present in QM. In order to understand the analogy with Einstein's fluctuation calculation in a thermal black body system, it is important to expose the mechanism which causes a global vacuum state to become impure on a localized subalgebra of QFT.

The present work presents the fascinating history behind this problem which culminated in the more recent perception that "causal localization" leads to thermal manifestations. The most appropriate concept which places this property of QFT into the forefront is "modular localization". These new developments in QFT led to a new access to the existence problem for interacting quantum fields whose solution has remained outside the range of renormalized perturbation theory. It also clarifies open problems about the relation of particles and fields in particular about the incompletely understood crossing property. Last not least it leads to a constructive understanding of integrable versus non-integrable QFTs..

1 QFT, how it begun and how modern concepts solve the Einstein-Jordan conundrum

It is well known that, long before the observational discovery of the photon, Einstein postulated a corpuscular nature of light [1] based on thermal fluctuation properties of black-body radiation. In a detailed theoretical analysis of subvolume fluctuations in a semiclassical (Bohr-Sommerfeld) statistical mechanics [2], he identified, in addition to the expected wave-like fluctuation component, a particle-like component which he interpreted as an indication of a corpuscular aspect of light. In his view the presence of this component was important for attaining thermodynamic equilibrium. Whereas the photoelectric effect constituted the first observational support for photons, Einstein's 1917 fluctuation result [2] which confirmed his earlier ideas [1] was of a purely theoretical kind. In view of the inaccessible nature of such fluctuations to direct observations, Einstein's argument remained a famous Gedankenexperiment in the setting of the old quantum theory.

This work attracted the attention of one of Max Born's younger collaborators who had some familiarity with statistical mechanics and first hand knowledge about the newly developing quantum theory. In his 1924 Ph.D. thesis [3] Pascual Jordan contradicted Einstein's assertion that one needs the presence of a particle-like component (Nadelstrahlung) to obtain thermodynamic equilibrium. Einstein's answer came swiftly; only some month later he published a counter paper [4] in which he showed that, despite the confirmed mathematical correctness of Jordan's thesis, there was a problem of a more physical nature on which he failed, namely the particle-like component was needed in order to get the right absorption coefficients.

This encounter with Einstein, which Jordan entered as an adversary of the theory of (what later was called) photons, did not only ruffle his feathers but, fortunate for the birth of particle theory, also put him onto his figurative "road to Damascus" in that he became the discoverer of quantum field theory (QFT), and the most uncompromising enunciator of a quantum theory (QT) of light *and* (de Broglie wave) matter within a unified setting of quantized fields.

There is a subtle irony in the fact that Jordan's radical change of mind was caused by Einstein, a lifelong opponent of quantum probabilities. If Jordan's claim of a complete analogy of quantum fluctuations in a subvolume with Einstein's thermal fluctuations really amounted to attribute a thermal aspect to his new wave quantization, then the new theory in its restriction to a subvolume should also admit an intrinsic probability namely that of thermal ensembles, with which Einstein was entirely familiar.

It can be assumed that neither Einstein nor Jordan were aware of these implication of the E-J "conundrum" [5]. As a result Jordan, in his dispute with Born and Heisenberg, did not receive the support of Einstein which he had hoped for. Looking back at this episode with historical hindsight, a great chance, which may have steered QFT from its very beginnings into a more foundational direction, was lost. Born's addition of a probability concept to

events caused by global observables[1] and states in quantum mechanics (QM), and some of the counter-intuitive aspects of QT would perhaps have appeared in a different light. Although there is no probability of thermal origin in QM, it is the more fundamental QFT which has the suzerainty over the conceptual problem about the nature of probability. In fact the main aim of the present work is to convince the reader that the concept of ensembles, which is inherent in Haag's presentation of QFT in terms of (subvolume) localized algebras, is the best setting for a complete resolution of the E-J conundrum.

In the famous "Dreimännerarbeit" with Born and Heisenberg [6], Jordan contributed a separate section containing a calculation of the mean square energy fluctuations in a subvolume for a simple model of quantized waves (2-dimensional "photons") applied to a two-dimensional free wave equation (see section 4); a model which he considered as a one-dimensional analog of Maxwell's theory of light. The result of his approximate calculation[2] consisted in the verification of the presence of wave and particle components in the subvolume fluctuation spectrum, just as in Einstein's statistical mechanics calculation, except that Jordan's global state was the vacuum state of QFT, whereas Einstein was analyzing subvolume fluctuations in a global thermal ("heat-bath") state in statistical mechanics. Jordan did not address the problem how subvolume (subinterval) fluctuations of a quantized system in a ground state can mimic those of a global thermal system; it is not even clear whether he realized that this is not possible in QM.

In the setting of Schrödinger quantum mechanics one knows that the global vacuum tensor-factorizes into the vacuum of a spatial subsystem and its complement[3]. Can the vacuum of QFT deviate from this quantum mechanical behavior i.e. could the local restriction of a global vacuum lead to thermal behavior? As a result of these kind of conceptual questions, which remained open in Jordan's contribution and were forgotten afterwards when QFT was on its path to success, the name *Einstein-Jordan conundrum* [5] is quite appropriate. In the present work it will be shown that the missing thermal properties of subvolume fluctuation in a global vacuum state of QFT can be verified and hence the conundrum aspect disappears. In Jordan's model of a two-dimensional wave equation, the analogy of subvolume-localized QFT with the thermal aspects of statistical mechanics amounts to an isomorphism of a global heat bath thermal system with a localization-caused thermal aspect of QFT (section 4).

Jordan's quantum theoretical calculation, which appeared as one section in the Dreimännerarbeit [6], did not receive the unrestricted endorsement of his coauthors Max Born and Werner Heisenberg [5]; too many assumptions in Jor-

[1] QM is a global quantum theory; there is no localization which leads to subalgebras (ensembles) of observables. Whereas the ensemble point of view in QFT and its thermo-probabilistic manifestantions is intrinsic, in QM it is up to the interpreter. This freedom has led to many heated disputes.

[2] Even though the system is noninteracting, subvolume fluctuations do not permit a computation in closed form.

[3] In order to facilitate a comparison of QM with QFT we will use throughout this article the "second quantized" form of QM for which the Schrödinger wave function is replaced by a quantum field.

dan's treatment of infinities and other unclear aspects in his approximation were swept underneath the rug and prevents them from embracing Jordan's remarkable but somewhat suspicious calculation. Whereas the calculation techniques of the new quantum mechanics were usually transparent, this was not the case in Jordan's field theoretical model.

Nowadays we know good reasons for such doubts; their conceptual origin is the clash between the *causal localization* of relativistically propagating QFTs and the much simpler localization structure of QM[4] based on the *Born localization* (resulting from the spectral theory of the quantum mechanical position operator). Whenever one has to rely on approximations, as in subvolume fluctuations of QFT, one must verify their compliance with causal localization.

It is well known that perturbative QFT led to serious problems with the formalism of QM for a long time until it was formulated in a *relativistic covariant* way (closely related to causal localization) at the end of the 40s. In fact the clearest formulation of relativistic perturbative theory results from the iterative implementation of the causality principle in the form of spacelike commutations of fields, which is known as the Epstein-Glaser approach [7]. In other formulations of perturbation theory using regularizations and cut-offs (which are often computationally more efficient) this is less clear. In addition there is the human aspect of forgetting the principles behind a formalism once it has been formulated in terms of efficient computational recipes.

In the case of the subvolume fluctuation problem the elegance of relativistic covariant perturbation theory is of no help; it is better to verify the consistency of an approximate calculation by using the closely related *causal localization* in a more direct manner. One of its manifestations is the use of the thermal KMS[5] [8] structure of the restricted vacuum state and to approximate this rather singular state by Gibbs density matrix states. Such problems are at best formulated and solved in the new *modular localization setting* (section 3).

Although there is a formal analogy to the thermodynamic limit in the heat bath (statistical mechanics of open systems) setting, the Hamiltonians of modular theory are generally not those which correspond to time translations of a non-inertial observers in Minkowski spacetime; in fact the automorphism of the localized algebra which they generate has generally no interpretation at all in terms of a *geometric* flow within the causally extended localization region (*fuzzy* automorphism). Beyond its preservation of the localization region, almost nothing is known for compact causal localization regions.

The theory which describes such fluctuation phenomena in a model-independent way is a special case of an abstract mathematical theory of operator algebras, which carries the name of its protagonists: the *Tomita-Takesaki modular oper-*

[4]Intuitively speaking this is the difference between infinite velocity propagation (action at a distance) and propagation with a limiting velocity. QM, whether formulated a la Schroedinger or second quantized, has no limiting velocity in its algebraic structure and any finite velocity as that of sound is "effective" i.e. arises in suitable expectation values for large times.

[5]The Kubo-Martin-Schwinger analytic characterization of thermodynamic limit states replaces the tracial Gibbs state (density matrix) formalism which breakes down as a result of volume divergence for V$\rightarrow \infty$.

ator theory [8]. In quantum physics one encounters this theory in two places: statistical mechanics (in particular in the Gibbs formulation and its thermodynamic limit), and in localization problems of QFT (such as that of the E-J conundrum). In this second role the setting is often referred to as *modular localization* [10]. Whereas the statistical quantum theory of open systems is most elegantly formulated in the setting T-T modular theory, the latter becomes really indispensable in the context of modular localization. It is the only way to describe the model-independent thermal nature of spacetime-restricted vacuum states in local quantum physics (LQP). Since a mathematically rigorous presentation of this setting would go beyond what one can reasonably expect of a reader with interest in the conceptual aspects of QFT to digest, we will sacrifice mathematical precision in favor of conceptual physical understanding. In this introduction and the next section some of the concepts will still retain their intuitive metaphoric meaning, only in the subsequent section some mathematical/conceptual precision will be added.

Even though a free quantum field obeying a linear wave equation can be adequately described in terms of global quantum mechanical oscillators[6] (momentum space creation/annihilation operators), this quantum mechanical description is not useful for QFT fluctuation problems in subvolumes. As mentioned the localization-induced thermal aspects of the E-J conundrum, which any approximate calculation should fulfill, are hard to reconcile with global quantum mechanical oscillator descriptions. More specifically, it is not clear how the global oscillators, in terms of which a free field can be written, can retain their utility in subvolume problems.

Spatial localization of quantum mechanical variable in terms of projectors associated with spatial regions which appear in the spectral decomposition of the position operator ("Born-localization") lead (after applying them to global states or operators) to Born-localized states or to local observables at a fixed time; the most convenient way to see this is a Fock space formulation of QM ("second quantization") which maintains the physical content, but brings QM into a formal analogy with QFT. In causal QFT such a spatially localized algebra is equal to the algebra localized in its spacetime causal completion which in the simplest case of a spatial ball is the double cone extended by the "causal shadow" with the ball as its base.

Whereas in QM the $\mathbf{x}$ ranges through the spectrum of the position operator, the points of Minkowski spacetime $\mathbf{x}$,t, which parametrize relativistic fields, have no such operator interpretation. Hence the quantum mechanical localization is directly linked to the probability interpretation which Born [11] added to Heisenberg's QM shortly after the Dreimännerarbeit. As a result of absence of a position operator in QFT[7], the Born probability looses its algebraic realization in terms of observables and continues to be important for wave functions[8] (vector

[6]The claim that a free field theory associated with particles of arbitraey spin "is nothing else that a collection of oscillators" is an exaggeration since a student of quantum mechanical oscillators would not be able to combine them into a covariant field.

[7]QFT in this article always refers to relativistic QFT.

[8]Even in QM its use for wave functions is physically more important than for second

states).

The difference in localization leads to significantly different mathematical structures and physical consequences. Algebras at equal times in the *Fock space* formulation of QM tensor-factorize into an $\mathcal{O}$-localized subalgebra and that localized in its spatial complement $\mathcal{O}'$. Operator algebras in QFT do not share this property, even though both algebras commute and together generate the global algebra. Related to this is the property of factorization of the nonrelativistic vacuum, whereas the QFT vacuum under subdivision becomes entangled in a very strong (singular KMS) sense associated with infinite vacuum polarization "clouds" at the causal boundary [13]. In fact local operator subalgebras in QM are with respect to their von Neumann type the same as their global counterpart namely isomorphic to $B(H),$ the operator algebra of all bounded operators on a Hilbert space H, and the Hilbert space suffers an inside/outside factorization $H = H_{inside} \otimes H_{outside}$ which follows the spatial split.

Local algebras in QFT are radically different, they are all isomorphic to an operator algebra which, for reasons which become clear later on, will be referred to as a *monad*. Saying that they act in a Hilbert space H, and therefore are subalgebras of $B(H)$, does neither help to understand their mathematical properties nor their physical role. In contrast to QM, a halfspace algebra at a fixed time (or its associated causally completed wedge-locaized algebra) and its opposite halfspace counterpart (causally disjoint wedge) commute but do not tensor-factorize (monads do not factorizes with their commutants). This leaves room for a very singular kind of entanglement which cannot be described in the standard setting of quantum information theory [38]. This kind of entanglement does not have to (and should not) be averaged over the "opposite" degrees of freedom; the associated probabilistic KMS state ("singular density matrix") is obtained just by subalgebra-restriction of the original global vacuum.

We promised the reader to refrain from damping his/her interest in conceptual historical problems reaching back to the dawn of QFT by presenting technical mathematical details. The only exception will be those cases for which technicalities admit a simple physical interpretation. One such case is that operator algebras in the context of LQP weakly closed i.e. they are von Neumann algebras. As the result of their algebraic characterization as consisting of subalgebras of $B(H)$ which remain preserved under the two-times application of forming commutants in $B(H)$, as well as the fact that commutants play an important role in the formulation of Einstein causality (statistical independence of spacelike separated measurements), the use of such algebras enjoys direct physical support. The local subalgebras of QFT are always factors (indecomposability).

Another difference from QM is the use of the word state and (state) vector. States are positive linear functionals ω on operator algebra i,e, $\omega(A),\ A \in \mathcal{A}$. Their physical meaning and the problem of their representation in terms of

quantized Schrödinger fields. Propagation velocities (e.g. the velocity of sound) describe the asymptotic movements of the position of maximal probability density of wave functions. Despite the frame dependence of Born localization its large time consequence for the movement of centers of relativistic wave packets in QFT comply with independence on inertiel frames.

vectors in a Hilbert space as $\omega(A) = (\psi, A\psi)$, $\psi \in H$, $A \in \mathcal{A}$ depend on the structure of the algebra. In QM where $\mathcal{A} = B(H)$, independent of whether $\mathcal{A}$ denotes a global algebra or a Born-localized subalgebra (in which case H is a subspece of the total Hilbert space), the state determines a vector uniquely up to a phase factor and therefore the identification of states with vectors in H makes good sense. This kind of uniqueness breaks down for other operator algebras which appear in the classification of factors, in particular for states on a monad. In cyclic representations (existence of a vector on which the application of the algebra creates a dense set in a Hilbert space) there is a unique relation between the algebra and a dense set of vectors, but the representation of states by vectors remains highly non-unique. The distinction between states and vectors is crucial in the study of localized subalgebras of QFT which are always of the algebraic monad structure.

This immense structural difference resulting from Born-localization in QM as compared to modular localization in QFT has been generally overlooked outside of LQP; in fact the latter may be understood as a formulation of QFT which highlights precisely these differences. An educated guess why QFT developed in this way is that QM and QFT share the formalism of Lagrangian quantization and functional integral representation and the important renormalized perturbation theory is based on computational recipes which do not place the antagonism of the underlying localization principles into sufficient evidence.

Whereas the functional integral approach is a rigorous mathematical tool in QM[9], the lack of its mathematical control in QFT is partially compensated by its intuitive suggestive content which together with some corrective hindsight often leads to correct recipes for perturbative calculations. As a result of the well established divergence of perturbative series, such calculations do not say anything about the existence of a model of QFT; but perturbation theory comes with a lower level of consistency (that of formal power series) which facilitates its extraction from mathematically nonexistent functional integrals with a modest amount of hindsight; it is not sufficient to show that the perturbative result admits a functional integral representation. Questions as to why in important cases the low terms of diverging power series lead to incredibly precise agreement with experimental data are not really answered by claiming that these power-series are asymptotically convergent in the limit of vanishing interaction strengths; as long as the existence of a model remains unproven such claims have no mathematical basis.

The problem of approximating the subvolume energy fluctuations in QFT is quite different since, at least in the absence of interactions, the existence is secured. However such problems cannot be solved without resorting to approximations, so the remaining task is to show that such approximations remain compatible with localization and its thermal aspects. In this respect the improved calculation in the Jordan model proposed in the work of Duncan and Janssen is somewhat contradictory because by claiming that the restricted vac-

[9]Even in QM it is not advisable to use functional integrals in a course on QM where exact solutions (integrability) are presented.

uum remains a pure state (see remarks after equ. (53) in [5]), one throws away the child with the bath tub[10]. Thinking in terms of QM on the other hand, it is natural to add a coupling to an external heat bath in order to enforce the thermal aspect (related to their belief that the subinterval restriction does not effect the purity of the restricted state) of the conundrum and this is precisely what Duncan and Jannsen did; but perhaps the vacuum really does not factorize in their approximation in which case there would only be a discrepancy between their (and Jordan's) possibly correct approximation with an incorrect verbal claim about factorization (which negates the thermal impure nature).

Often physicists use loose language by calling QFT "(relativistic) quantum mechanics". This incorrect terminology can create conceptual havoc in those cases in which the emphasis on the differences becomes essential. To remain clear on this point, it may be useful to mention that relativistic quantum mechanics as being something different from QFT really exists; it is known under the name *direct particle interactions* [12] and describes a theory which is solely formulated in terms of particles and their Poincaré invariant scattering matrix (no covariant local observables) and has no conceptual relation with QFT [13].

Many articles and books create the impression that the mere existence of infinite degrees of freedom separates QFT from QFT. But as the existence of a second quantized presentation of QM shows, this is not the case. What is however true is that in QM the appropriately defined phase space density (degrees of freedom per unit cell of phase space) is finite, whereas the causal localization of QFT requires a "mildly" infinite ("nuclear") phase space degree of freedom behavior [8].

It is very difficult to check by hand within the standard setting of QFT whether a calculation is consistent with thermal aspects of subvolume fluctuations; it is easier to use a formulation of QFT which takes the thermal aspect into account from the outset. An adequate setting which guaranties that thermal aspects of localization are correctly implemented can be given in the setting of *local quantum physics* (LQP), also referred as *algebraic quantum field theory* (AQFT); such a setting dates back to a seminal 1957 talk by Haag (for a recent translation see [14]). From its humble beginnings it has developed into a nonperturbative mathematically precise intrinsic setting of QFT i.e. a formulation of QFT which does not depend on a quantization parallelism to classical field theory (last section); in particular it does not require to understand how thermal aspects, which are absent on the classical level, emerge through quantization.

The conceptual progress of QFT has revealed that Haag's intuitive idea of localization in terms of local observables, envisaged as counters which have a finite extension in space and are switched on for a finite duration in time, leads to unexpected somewhat metaphoric situations if its exact mathematical formulation is re-interpreted in terms of a Gedankenexperiment. Even in the simplest of all cases, the noncompact localization in Rindler wedges of Minkowski spacetime, the appearance of a measurable thermal radiation in the Unruh Gedankenexper-

[10] Since this is a common conceptual misunderstanding of QFT, the criticism is not personal; in fact it is probably the way Jordan considered his subvolume fluctuations.

iment (see later) requires to uniformly accelerate the hardware with absurdly big acceleration which cannot be achieved with macroscopic counters. Such Gedankenexperiments reveal an unexpected side of localization. They focus attention to an aspect of QFT which, although somewhere hidden in the Lagrangian quantization setting, is naturally accounted for in the LQP formulation of QFT.

There is hardly any conceptual enrichment of QFT which has been as fruitful for this kind of problems as Haag's algebraic LQP setting which describes the model independent nature of such phenomena. Direct attempts at physical realizations of principles in form of Gedankenexperiments may acquire a somewhat metaphoric counter-intuitive appearance (perhaps the reason why the Unruh effect has led to many controversies), but as long as their mathematical formulation is precise and sufficiently many (possibly indirect) physical consequences agree with observational tests a theory is successful.

The strategy underlying the LQP setting is in a way opposite to that of quantization which is based on analogies to classical physics which, with the exception of QM remain mathematically vague and whose underlying physical principles cannot easily be seen from the computed perturbative results. On the other hand in LQP one first formulates the principles and properties which a physically acceptable QFT should have in a mathematically rigorous way; only afterwards one looks for methods to classify and construct models which fulfill these "axioms" and comply with experimental observations (top-to-bottom approach).

It is the main aim of the present work to explain these properties and show how the Einstein-Jordan conundrum, including its thermal aspects, can be understood. For Jordan's model of a two-dimensional zero mass wave equation the relation is explicit and amounts to an isomorphism of the two systems (section 4). The foundational aspects of QFT were present since its beginnings in 1925, but their understanding in the ongoing research is still a project which, different to QM, had yet not arrived at a conceptual closure.

Jordan's coauthors Born and Heisenberg felt that his presentation of subvolume fluctuations of quantized waves did not quite fit into the quantum mechanical setting of their joint work, which was the second paper written after Heisenberg's presentation of QM. They did not see that Jordan with his quantization of waves discovered the beginnings of a new QT which went beyond the scope of QM. In a letter Heisenberg challenged Jordan several years later [5] to account for a term which diverges proportional to $log\varepsilon^{-1}$ and which Jordan should have seen in his two-dimensional model of quantized waves; here ε is a measure of "fuzzyness" at the endpoints of the localization interval. This shows that the differences to QM of Jordan's wave quantization was slowly being appreciated.

This correspondence preceded Heisenberg's famous paper on vacuum polarization [15]; it represents the diverging contribution from vacuum polarization in the limit of sharp localization. In his paper Heisenberg implicitly proposed that for models in 4-dimensional spacetime vacuum polarization caused by localizing dimensionless observables leads to a divergence proportional to the dimension-

less "fuzzy" surface A/ε^2. He exemplified this in the case of a dimensionless *partial* charge localized in a finite volume and showed that the formation of particle/antiparticle pairs near the surface is the cause of this behavior. It has no counterpart in classical field theory and in QM. The distribution theoretical setting which permits a rigorous derivation of this behavior of partial charges from the singular properties of their pointlike conserved currents became available only after it was realized that relativistic covariant fields and currents were operator-valued distributions [16] i.e. objects which had to be smeared with test functions before they became (generally unbounded) operators. This will be briefly sketched in the next section.

Heisenberg's vacuum fluctuations are closely related to the thermal aspects of localization, but it is not so easy to see this in the standard quantization formalism. The setting of Haag's LQP, in which the concept of modular localization of states and operators permits a natural and precise formulation is therefore better suited for this purpose. It permits to explore the powerful Tomita-Takesaki modular operator theory for the localization problems of QFT.

The first step in linking QFT with the T-T modular theory was the realization that the *statistical mechanics of open systems*, i.e. the direct description of thermal states in the infinite volume limit, is a special case of the T-T modular theory. The limiting states loose their characterization as density matrix Gibbs states; what remains is their KMS property which, before it became the defining property of thermodynamic limit states, was used in the work of Kubo, Martin and Schwinger as an analytic tool which facilitated the calculation of traces of Gibbs density matrices. In a seminal paper by Haag, Hugenholtz and Winnink published in 1967, this observation was elevated to a fundamental property which follows from the stability requirements of thermodynamic equilibrium [8].

It was a lucky coincidence that physicists working with operator algebraic methods met rather early (at the 1967 international conference in Baton Rouge, see [48]) with mathematicians who already had obtained important results on operator algebras which generalized what had been obtained from the study of (unimodular) Haar measures within group representation theory and became referred to as "modular operator theory". In this way both theories were combined; the mathematicians incorporated the ideas around the conceptual use of KMS and the physicists adopted the Tomita S-operator and the (Tomita-Takesaki) modular theory.

The important point in the present context is that the algebra changed its nature in the thermodynamic limit; whereas the approximating box-quantized statistic mechanic algebras are of type I[11] i.e. isomorphic to the $B(H)$ global algebras (both in QM/QFT), the open system (thermodynamic limit) algebra is (in QM and QFT) a hyperfinite factor algebra of type III_1 in Connes extension of the Murray-von Neumann classification. In fact the KMS property provided by modular operator theory played an important role in Connes refinement of classification of factor algebras [17]. More recently there were attempts to

[11] This is the standard algebra encountered in QM consisting of all bounded operators B(H) of a Hilbert space H.

interpret time as originating from the operator algebraic KMS property [18] which is opposite to relating proper time with (the Unruh) temperature.

The property which secures the applicability of that theory is the *standardness* of the thermal operator algebra i.e. that the existence of a vector Ω representing a thermal state in the Hilbert space H on which the application of the operators A of the algebra $\mathcal{A}$ generate a dense set of states (cyclicity) and does not contain nontrivial annihilation operators:

$$\begin{aligned} &the\ subspace\ \mathcal{A}\Omega\ is\ dense\ in\ H\ (cyclic) \\ &if\ A\Omega = 0\ for\ A \in \mathcal{A} \subset B(H) \curvearrowright A = 0\ (separating) \end{aligned} \tag{1}$$

where $B(\mathcal{H})$ denotes the algebra of all bounded operators. The T-T operator theory associates with such a standard pair a densely defined involutive antilinear Tomita S-operator

$$\begin{aligned} &SA\Omega \equiv A^*\Omega,\ S^2 = 1\ on\ its\ domain\ domS \\ &S = J\Delta^{\frac{1}{2}} = \Delta^{-\frac{1}{2}}J, \end{aligned} \tag{2}$$

for which its closure (denoted with the same letter) has the polar decomposition (written in the second line) in terms of antiunitary reflection J and a positive operator Δ which leads to the unitary modular group Δ^{it}. Whereas the proof of the above properties is rather straightforward, proving the T-T theorem is anything but simple [35]. It states that J maps $\mathcal{A}$ into its algebraic commutant $\mathcal{A}'$ (the algebra of operators in $B(H)$ which commute with every operator in $\mathcal{A}$), and Δ^{it} defines a modular automorphism σ_t of $\mathcal{A}$

$$\begin{aligned} &J\mathcal{A}J = \mathcal{A}',\ \sigma_t(A) = \Delta^{it}A\Delta^{-it} \in \mathcal{A}\ for\ A \in \mathcal{A},\ \Delta^{it} = e^{-itH_{mod}} \\ &\omega(A) \equiv (\Omega, A\Omega)\,,\ KMS:\ \omega(A_1A_2) = \omega(A_2 e^{-H_{mod}}A_1),\ A_i \in \mathcal{A} \end{aligned} \tag{3}$$

where the introduction of the modular Hamiltonian serves to show that the modular KMS in modular theory the dimensionless "temperature" corresponds to $\beta = 1$. For more details of the description of equilibrium statistical mechanics in this setting the reader is referred to [8].

The presentation of the subsequent sections is facilitated by adding some more notation and comments on its physical meaning. Although *global* algebras in ground state problems (in contrast to thermal states) of both QM and QFT lead to the same type of algebras of all bounded operators in a Hilbert space $B(H)$, there can be nothing more different than the result of passing to localized subsystem. In QM the spatially localized subalgebras remain $B(H_{sub})$ algebras where $H_{sub} = PH$ with P a projector from the spectral decomposition of $\mathbf{x}$ associated with that part of the spectrum which corresponds to the localization region $\mathcal{C}$

$$\begin{aligned} &nonrel.:\ B(\mathcal{C}) = B(H(\mathcal{C})),\ type\ I_\infty \\ &QFT: \mathcal{A}(\mathcal{O}),\ hyperfinite\ type\ III_1 = \text{"}monad\text{"} \end{aligned} \tag{4}$$

whereas in the QFT case it undergoes a radical change in that all the localized algebras are of the type of a monad independent of the localization region. "Monad" is a short hand terminology for an isomorphy class of indecomposable representations of a unique von Neumann factor algebra (hyperfinite type III_1 factor algebra) where the factor property replaces the irreducibility of $B(H)$. The timelike causal shadow property (see (9) next section) permits to replace a simply connected spatial localization $\mathcal{C}$ by the spacetime localization in the associated causal shadow $\mathcal{C} \rightarrow \mathcal{C}''$. Algebras which result from sharp localization in QFT are always isomorphic to a monad; only suitably (*split-property* [19]) defined "fuzzy"-localized algebras can be of type $B(H(\mathcal{O}_{f.l.})$ with $\mathcal{O}_{f.l.}$ the spacetime region of fuzzy localization.

By using the terminology "monad" we do not expect the reader to know the classification of operator algebras; for a physicist it is more important to understand a monad through its physical properties [20]; some of them will appear in the next three sections. Sharp causal localization cannot be expressed in terms of projectors; localization in terms of projectors and (sharp) causal localization are mutually exclusive. The most surprising if not spectacular physical property of a monad is that a full QFT (including its Poincaré group acting on a d-dimensional Minkowski spacetime) can be encoded into an abstract Hilbert space positioning of a finite number of copies of a monad without any internal structure. This attributes to QFT an ultra-relational aspect which no other type von Neumann factor is capable of generating (see later).

Although some physicists with an early familiarity with LQP probably knew that, as a result of the Reeh-Schlieder property of localized subalgebras in QFT the standardness of a spacetime localized algebra $\mathcal{A}(\mathcal{O})$ with respect to the vacuum and therefore the applicability of the T-T modular theory always holds, the awareness about its physical implications had to wait another decade. It came in a paper by Bisognano and Wichmann [21] when these authors realized that the above modular objects in the case of wedge algebras have a geometric physical interpretation: the unitary modular group is given in terms of the operators representing the wedge-preserving Lorentz boosts and the J is the antiunitary operator which reflects on the edge of the wedge (the TCP operator up to a π-rotation). A special $z-t$ wedge region is defined as $W_{z,t} = \left\{ z > |t| \, ; \; x, y \in \mathbb{R}^2 \right\}$ and a general wedge is obtained by applying Poincaré tranformations.

The proof given by B-W, which leads to the mentioned geometric identification of the modular objects, still depended on some reasonable technical assumption about operator algebra properties resulting from quantum fields. A direct proof in the algebraic setting of LQP can be found in [22]; this proof is free of technical assumptions and uses besides the requirements of LQP the physically motivated property of the validity of a complete particle interpretation. In this way the modular wedge localization of Wigner wave functions [23] turns out to be useful for proving its interacting algebraic counterpart.

In general the modular objects do not act geometrically in other cases, but the modular unitaries always respect the causal boundaries of localization and the local algebras can be defined (if necessary by "dualization") in such a way that the commutant is equal to the algebra associated to the causal complement

(Haag duality [8]). The modular unitaries of other regions $\mathcal{O} = \cap_{W \supset \mathcal{O}} W$ which allow a geometric representation in terms of intersections of wedges, the modular data of $\mathcal{A}(\mathcal{O})$ are geometric in the indirect sense of being determined in terms of the modular data of the participating intersecting wedge algebras.

Since the unitary modular groups of wedges are the unitaries of the wedge-preserving Lorentz boosts and the representation of the Poincaré group is shared between the interacting theory and its free field asymptote, the only dependence on the interaction can be in J. Indeed the interacting J is connected with its free field counterpart J_0 via the scattering matrix $J = S_{scat} J_0$ [10]. The conceptual simplicity of the definitions stands however in stark contrast to the difficulties one encounters in attempts to calculate the modular data of intersections.

This concept of modular localization leads to thermal manifestations of localization in terms of modular KMS properties. They share some properties with statistical mechanics (localization-caused versus heat bath thermal behavior). A pure global vacuum state reduced to a spacetime subregion becomes a highly entangled state with a rather singular notion of entanglement which is shared by all monad representation independent of their origin. The sharply localized reduced state ω cannot be described in terms of a density matrix inasmuch as a density matrix Gibbs state looses this property in the thermodynamic limit. Despite the shared KMS property between the "heat bath thermality" of statistical mechanics and the "localization thermality" of QFT there are also important differences. Localization thermality is more abstract because the modular Hamiltonian is never related to the translation in Minkowski spacetime, it is rather intrinsically determined by the standard pair $(\mathcal{A}(\mathcal{O}), \Omega)$.

As a consequence the modular analog of temperature cannot be directly measured, its main physical purpose is to describe the singular (i.e. not density matrices) impurity of subvolume-reduced vacua. In the last section this theory will be applied to Jordan's model. This belongs to the class of conformal QFTs which have admit geometric modular groups for certain compact regions which arise by applying conformal maps to wedges. In the case of Jordan's chiral conformal model the relation between an Einstein statistical mechanics and the restricted vacuum QFT is an isomorphism. Such special situation have been referred to as an "inverse Unruh effect" [24]. It is believed that this is restricted to two dimensions.

The historical remarks about thermal aspects of localization would remain incomplete without commenting on the *Hawking radiation*. In that case the localization results from a reduction of a Hartle-Hawking state on the global Kruskal extension of a Schwarzschild spacetime to the region outside the black hole event horizon; the modular Hamiltonian is expected to be proportional to that of the time-like Killing flow in that region. In the literature this connection with localized quantum matter is generally not mentioned; the thermal aspect is often viewed as a thermal manifestation which can only occur in curved spacetime. Whereas it is certainly true that the formation of black holes from collapsing stars and the onset of Hawking radiation is a phenomenon within general relativity, the resulting thermodynamic equilibrium at the Hawking temperature is a special case of a thermal manifestation of localization, namely localization

of quantum matter outside a black hole.

The main difference to the B-W situation is that, whereas the Killing time is the only natural (observer independent) time (corresponding to the Minkowski time in the zero curvature case), the infinitesimal generator of the wedge-preserving $z-t$ Lorentz boost is proportional to a Hamiltonian associated to the proper of a uniformly accelerated observer (radiation counter) in the z-direction. This is Unruh's realization [25] of localization within a Rindler-wedge, which, as the result of the impossibility to accelerate counters to an extend for obtaining a measurable temperature, remained a "Gedankenexperiment" similar to the E-J conundrum. The connection of thermal manifestations of quantum matter behind causal and event horizons with modular theory was given in an important paper of Sewell [26].

The intuitively simple identification of a compact spacetime-localized observable pictured in the LQP framework as being related to a measurement in a spatial region with finite duration looses its simplicity; if one tries to re-express the mathematical description of an ensemble of observables in terms of physical hardware (radiation counters) in analogy to the Lorentz boost Hamiltonian of the Unruh effect the intuitive aspect of modular localization becomes somewhat metaphoric. A concrete realization of a Hamiltonian for compact localization appears to be impossible. This shows that an intuitively clear concept, as Haag's localized observables, may take on a metaphoric appearance if one asks detailed questions about its precise realization in spacetime.

As mentioned before, as long as the mathematical formulation of physical principles is precise and measurable consequences agree with predictions, such changes of intuitive arguments under close conceptional scrutiny are of no concern. QFT is certainly the most successful foundational theory of quantum matter, but it is also the theory which still contains the largest number of conceptual surprises. The thermal aspects of the E-J conundrum, which is the clearest indication that Jordan really discovered a new kind of QT (and not a relativistic form of QM), is a good illustration.

It has become fashionable to use the word *holistic* for those properties which set QFT apart from QM in the Fock space formulation [27]. The strongest illustration of the holistic aspects of QFT is the existence of an encoding of a full QFT (including its Poincaré covariance acting on Minkowski spacetime as well as its inner symmetries) into an appropriate relative positioning of a finite number of copies of a rather structureless monad which act in a shared Hilbert space. Never before in the history of theoretical physics has a theory been "relational" to such an extreme degree as in this characterization of QFT. The analogy of such a presentation with Leibniz's philosophical attempt to understand reality as resulting from the interplay of impartible objects which he called *monads* suggests to use this terminology also in the present context (instead of the rather lengthy name within a classification theory of indecomposable von Neumann algebras: "hyperfinite type III_1 factor algebras"). This is in particular supported by the fact that QFT deals with only two types: sharply localized monads and fuzzy localized $B(\mathcal{O}_{f.l})$ (as well as global algebras $B(H)$). QM (at zero temperature) on the other hand only uses the $B(H)$ type. The positioning

needed in this construction of a model of QFT is called "modular" (modular inclusions, modular intersections) [28]. The Poincaré covariance is generated from the individual modular groups associated with each monad together with the vacuum state. The inner symmetry is encoded in the superselection sector determined by the representation classes of the observable net of algebras [29]

Such an encoding of spacetime aspects into a Hilbert space positioning is impossible in QM; the quantum mechanical localization and the associated probability is not intrinsic, it has been added to the quantization rules as an interpretative indispensable tool by Max Born, a step which caused Einstein's lifelong philosophical dissatisfaction with the post Bohr-Sommerfeld formulation of QT.

The terminology "holistic" is primarily used in connection with organic matter. Saying that living things consist of water and certain chemicals is of little help to understand how they work. In the case of Jordan's fluctuation model the decomposition into oscillator modes is not wrong but risky because one is inclined to use approximations which violate the holistic aspect of QFT without being aware of this; especially if, as in [5], one implicitly enforces a quantum mechanical setting by stating that the restricted vacuum remains a pure state. Quantum mechanical computational techniques applied to infinitely many oscillators are always in danger to violate the holistic aspects of causal localization. The mathematical raw material (as the shared Fock space and the momentum space creation/annihilation operators) may be the same; but important are not the quantum mechanical oscillators themselves, but rather the resulting holistic object which is obtained with or without their use. Whereas for free fields and perturbation theory the holistic aspect has been explicitly worked into the formalism, this cannot be said about the E-J fluctuation problem.

This has been pointed at in different contexts by other authors. For example Ehlers (cited in [5]) conjectures that Jordan's fluctuation problem is intimately related to unsolved aspects in the application of QFT to the problem of the cosmological constant. Hollands and Wald are quite outspoken on this issue when they say: "Quantum Field Theory Is Not Merely Quantum Mechanics Applied to Low Energy Effective Degrees of Freedom" [30]. They use the Casimir effect for their illustration. Additional remarks about QFT's holistic aspects can be found in the last section.

Jordan may have had a premonition that QFT cannot be subsumed under "QM with infinite degrees of freedom", but his quantized wave setting was too imprecise for such distinctions. Knowing these properties with the hindsight of 70 years of conceptual development in QT, one is inclined to view the critical position of Jordan's coauthors in a milder light; this was not a fight of a radical mind against his conservative detractors.

Jordan could not have acquired the status of the *unsung hero of QFT,* which he earned according to the opinion of historians and philosophers of physics [32][33], if he would have merely proposed an extension of QM to infinite degrees of freedom. In that case he may have earned the unrestricted support of his collaborators Born and Heisenberg since the bulk of their joint "Dreimännerarbeit" deals with problems of the newly discovered QM, but historians may not have considered him the protagonist of QFT.

The paper is organized as follows. In the next section Heisenberg's area law of vacuum fluctuations resulting from localized charges is presented in a contemporary setting of QFT. The third section provides some details on modular localization of states and operators. In the last section it is shown that by using the conformal invariance of Jordan's two-dimensional illustration, in this case the relation to a heat bath thermal system is not only an analogy but even an isomorphism. The paper concludes with a brief critical review at developments in which the issue of causal localization was misunderstood as well as some speculative remarks about future developments of QFT.

In order to keep a check on the length of the bibliography, we refer wherever this is possible to Haag's book on "Local Quantum Physics". Therefore citations in the present paper are not directly reflecting the prominence of their protagonists.

2 Heisenberg and the localization-caused vacuum polarization

Of his two coauthors in the Dreimännerarbeit, Heisenberg presented the strongest resistance against Jordan's claim of the solution of the energy fluctuation in open subvolume [5]. He felt that there were too many loose ends and uncontrolled assumptions. As time passed, Heisenberg became more articulate. Beginning in the early 30s he became increasingly aware of a characteristic phenomenon of QFT, which for composite fields is even present (although in a milder form) in the absence of interactions: *vacuum polarization.* Of course the knowledge that the vacuum polarization-caused impurity of a subvolume-restricted vacuum and its thermal manifestations go together was not available at that time.

Heisenberg was probably the first who thought that the omnipresence of vacuum polarization on sharp localization boundaries (endpoints of intervals in Jordan's simplified chiral current model) create infinities which are only controllable by making the boundary in some intuitive sense "fuzzy". Nowadays there is a very precise setting in which this problem allows a rigorous formulation: the distributional aspect if pointlike fields and the algebraic *split property*[12] [8]. At the beginning vacuum polarization was understood as a general consequence of systems with infinite degrees of freedom, but afterwards it was noticed that its appearance is specific for the causal localization property of QFT; the quantum mechanical vacuum remains inert even in the limit of infinite particle number $N \rightarrow \infty$. It may not be obvious to the untrained eye, but the localization in QM (in second quantization form), where Schrödinger creation and annihilation operators do not appear together in one object (and where the classical velocity is infinite[13]), is radically different from the causal localization of QFT (for which the limiting propagation speed c is defined algebraically in terms of the causal

[12] It would be interesting to compare the split property and its resulting area proportionality of entropy to A/ε^2 with ε the size of the split with 't Hooft's more speculative brickwall [34] idea which he uses to derive the Bekenstein area law.

[13] The mean velocity of wave packets is finite (the acoustic velocity in solids).

commutation relations). Holistic properties do not force an extension of QFT, they only require a change in our view of QFT.

The simplest illustration of a comparison in terms of shared oscillator-like momentum space annihilation/creation operators looks as follows

$$a_{QM}(\mathbf{x},t) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int e^{i\mathbf{px}-it\frac{\mathbf{p}^2}{2m}} a(\mathbf{p}) d^3p, \;\; [a(\mathbf{p}), a^*(\mathbf{p}')] = \delta^3(\mathbf{p}-\mathbf{p}') \tag{5}$$

$$A_{QFT}(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int \left(e^{-ipx} a(\mathbf{p}) + e^{ipx} a^*(\mathbf{p})\right) \frac{d^3p}{\sqrt{2p_0}}, \;\; p_0 = \sqrt{\mathbf{p}^2 + m^2}$$

The global operator algebra generated by smearing with testfunctions of unrestricted support is in both cases the same, namely the algebra of all bounded operators $B(H)$, but nothing could be more different than the local algebras generated with test function of localized support in a compact t=0 region[14] $\mathcal{C}$. Whereas in the case of QM the generated operator algebra remains of the same type $B(H(\mathcal{C}))$ (called type I_∞ von Neumann factor in a systematic classification of all von Neumann factor algebras), the local algebras of QFT $\mathcal{A}(\mathcal{O})$ defined in terms of the causal completion $\mathcal{O}''$ are factor algebras of hyperfinite type III_1 which, for reasons explained in the previous section, will be called *monad*. The only place in the QM setting where a monad appears is in the thermodynamic limit of thermal Gibbs systems [35].

Although the difference in (5) appears to be small and consist in a different Fourier transform of the $a(\mathbf{p})^\#$ with a different energy dependence as well as the appearance of both frequencies in the case of QFT, and even though the relative equal time commutator effectively limits the relative nonlocality between the two fields to the size of the Compton wave length, the consequences of the structural differences are enormous. The restriction of the vacuum to localized operator algebras of QFT is an impure KMS state similar to the thermodynamic limit state of QM coupled to a heat bath, even though in this case there was no coupling to a heat bath. This property will be presented in more details in the next section.

Another related aspect is the appearance of a vacuum polarization at the localization boundary of the causal completion (the *causal horizon*). This phenomenon can also be seen in the behavior of individual operators; they obey a KMS relation which has no counterpart in QM. Even for free fields this happens. If one passes to composites as e.g. to a conserved vector currents or energy-momentum tensor associated with a free field the vacuum polarization can be seen directly. In this way it was first notice by Heisenberg [15]. In fact he correctly guessed already in 1931 in a private correspondence (see [5]) that Jordan may have missed a $\log\varepsilon$ contribution where ε is the "security distance" around the end points of the localization interval (which has to be there in order that the vacuum polarization can "attenuate" and in this way unphysical ultraviolet divergences can be avoided).

[14] As a result of causal relativistic propagation the QFT algebra is automatically determined in the causal shadow (causal completion) $\mathcal{O}''$.

Nowadays such a "fuzzy collar" around a sharp boundary is automatically taken care of by saying that fields are Schwartz distributions so that smearing functions which are characteristic functions of a localization regions are ruled out. This formalism is however not applicable to the localization of operator algebras, in that case one has to refer to the previously mentioned split property which also unravel the thermal side of localization.

It is interesting to follow the steps which led Heisenberg to vacuum polarization in localized operators from a modern[15] viewpoint. In the classical setting a conserved charge is the space integral over a conserved current

$$\partial^\mu j_\mu = 0,\ Q_V^{clas}(t) = \int_V d^3x\ j_0^{clas}(t,\mathbf{x}) \tag{6}$$
$$Q_V^{QM}(t) = \int_V d^3x\ j_0^{QM}(t,\mathbf{x}),\ Q_V^{QM}(t)\Omega^{QM} = 0$$

The partial charge in the volume V is still t-dependent and becomes a dimensionless time-independent constant in the limit $V \to \infty$. The partial charge in second quantized charged Schroedinger QM can be defined in the same way apart from the fact that $Q_V(t)$ is now an *operator* which annihilates the quantum mechanical vacuum Ω^{QM}. The situation changes radically in QFT where this way of writing does not make sense because quantum fields are by there very nature rather singular object (they are operator-valued distributions). The degree of singularity follows in this case from the property that a charge must be dimensionless ($d(Q) = 0$) and hence $d(j_\mu) = 3$. In scale-invariant theories this would fix the inverse short distance power in x, whereas in massive theories it determines only the short distance singular behavior of conserved currents.

Heisenberg's important observation which led to the terminology "vacuum polarization" was made formally, i.e. without the use of test functions (distribution theory came two decades later) on "partial" charges associated to conserved currents of charged (complex) relativistic free fields (s=0,1/2). The conserved current is a charge neutral bilinear local composite which, apart from the Wick-ordering of the involved product of free fields, is equal to the classical Noether expression, and its application to the vacuum creates neutral pairs of particles (hence the term "vacuum polarization"); in the presence of interactions their number is always unlimited ("polarization clouds").

Heisenberg found that if one integrates the zero component of the conserved current of a charged free field over a finite spatial region of radius R, the so defined "partial charge" diverges as a result of vacuum polarization at the boundary, and that only by integrating all the way to infinity one obtains a finite polarization-free global charge of the respective state on which the operator is applied.

In the modern QFT setting it is possible to control the strength of such a divergence in terms of specially prepared test functions. Elementary and composite fields are singular but very precisely defined objects, they are operator-

[15] Here modern means Schwartz distribution theory, which gave a solid mathematical structure to the notion of singular quantum fields.

valued distributions. Whereas in QM the knowledge of the use of the Dirac delta functions suffices, the correlation functions of fields in QFT and their handlings necessitate at least a rudimentary knowledge of Schwartz distribution theory which exists since the beginning of the 50s and played an important role in Wightman's approach to QFT. A finite partial charge in n spacetime dimensions with a vacuum polarization cloud within a spherical region of thickness ΔR is is defined in terms of the following *dimensionless* operator

$$Q(f_{R,\Delta R},g_T)=\int j_0(\mathbf{x},t)f_{R,\Delta R}(\mathbf{x})g_T(t)d\mathbf{x}dt,\ lim_{R\to\infty}Q(f_{R,\Delta R},g_T)=Q \quad (7)$$

$$\|Q(f_{R,\Delta R},g_T)\Omega\|\equiv F(R,\Delta R)\overset{\Delta R\to 0}{\sim}\begin{cases}C_n(\frac{R}{\Delta R})^{n-2}\ for\ n>2\\ C\ ln(\frac{R}{\Delta R})\ for\ n=2\end{cases}$$

where the spatial smearing is in terms of a test function $f_{R,\Delta R}(\mathbf{x})$ which is equal to one inside a sphere of radius R and zero outside $R+\Delta R$, with a smooth transition in between; and g_T is a finite support $[-T,T]$ interpolation of the delta function. As a result of current conservation such expressions converge[16] with this special choice of "smearing" functions for $R\to\infty$ independent of ε to the global charge either weakly [36] or (of one sends T together with R appropriately to infinity) even strongly on a dense set of states [37]. This aspect of the quantum Noether issue is quite important because in addition to the "normal" behavior there are two other cases which have no classical or quantum mechanical counterpart.

The first case which has been exemplified by Goldstone leads to a divergence of the partial charge in the limit of $R\to\infty$ due to a zero mass Goldstone particle [8] which couples to the conserved current and prevents the convergence to a finite limit and is referred to as "spontaneous symmetry breaking". The second case is the Schwinger-Higgs "charge screening" in which the sequence of partial charges $Q(f_{R,\Delta R},g_T)$ converges to zero; in this case there is no (charge) symmetry to start with which can be broken. This is important for describe massive vectormesons in the setting of renormalized nonabelian gauge theory [38]. Often the Schwinger-Higgs mechanism is erroneously referred to as a symmetry breaking (which symmetry? gauge transformations are not physical symmetry transformations the transition from charge conservation to zero charge can hardly be called a symmetry breaking). A theorem relates the transition from massless to the massive vectormeson with the charge screening [36][39].

For an estimate of the vacuum polarization one is interested in the limit of $\Delta R\to 0$ for fixed R of $F(R,\Delta R)$ (7). As expected and already argued by Heisenberg, the computation of the partial charge in Jordan's chiral conformal model shows a logarithmically divergent behavior, whereas for n-dimensional models in case of n>2 the vacuum fluctuations in the fuzzy boundary are proportional to the "dimensionless area" $\frac{area}{(\Delta R)^{n-2}}$ which diverges in the limit of

[16] However inside commutators with other localized operators the partial charge is already time-independent as soon as their causally completed localization region is contained in that if the partial charge. In this algebraic sense the partial charge is really "partially" time-independent.

sharp localization $\Delta R \rightarrow 0$ of the partial charge as in (7). The calculation is particular simple in the massless conformal limit of a conserved current. The same limiting behavior which appears in the dimensionless partial charge also shows up as the leading short distance terms in the (also dimensionless) *localization entropy* [40] which refers to a fuzzy localized operator algebra instead of a single operator (next section).

The correct treatment of the perturbative vacuum polarization contributions was a painful process which almost led to the abandonment of QFT (the ultraviolet crisis of QFT). The importance of causal localization as the holistic principle which separates QFT from (infinite degree of freedom) QM and has to be upheld even in approximations was certainly not known at that time. Eventually a formulation was found which avoids intermediate violations of locality and Hilbert space requirements (positivity) caused by the cutoffs and regulators of the old renormalization method and instead presents renormalized perturbation theory directly in terms of its foundational root as an iterative implementation of the causal localization principle together with the requirement of a maximal scaling degree[17]. As an implementation of a principle, this Epstein-Glaser approach [7] is free of any intermediate ultraviolet divergences resulting from a hidden incorrect handling of vacuum polarization which plagued older formulations.

It is believed that among all interacting models which allow a characterization in terms of Lagrangian presentation, only the renormalizable models have the chance to be supported by a future mathematical existence proof. But since the perturbative series diverge, this has remained one of the great unsolved problems which has no counterpart in any other area of theoretical physics (where it was always possible to find mathematically controlled approximations). In this connection Gedankenexperiments as the Einstein-Jordan conundrum (or Unruh's localization in the Rindler wedge in terms of accelerated observers) are valuable because they point towards another face of QFT which, even after almost a century, simply remained outside the range of the standard quantization formalism. Hence such Gedankenexperiments may reveal good reasons why our conceptual mathematical access remained insufficient for establishing the mathematical existence of quantum models behind classical Lagrangians.

The heuristic aspects of Lagrangian quantization and functional integral representation are sufficient for starting renormalized perturbation theory, but they do neither help in conceptual mathematical problems of establishing existence nor are they even useful to understand thermal and entropic aspects of localization. Problems as the Einstein-Jordan conundrum remind us of unfinished business which holds QFT back from its closure.

The standard model of a QFT is one with a complete particle interpretation i.e. one in which fields are related via large time scattering asymptotes to particles and the full Hilbert space is a Wigner Fock space generated by those in/out particles. In such models the fact that particles are directly related to

[17]The bound on the scaling degree can only be fulfilled if the lowest order interaction in terms of local Wickproducts of free fields has scaling degree ≤ 4. The resulting finite parametric expression defines an island in the infinite parametric result (obtained without the scaling restriction) which is left invariant under renormalization group transformations.

measurable quantities as scatting amplitudes and the closely related formfactors is of considerable practical and theoretical value. An intuitive argument, which relates properties of formfactors which are the n-particle components of a state $A\left|0\right\rangle$ obtained by "banging"[18] with a local operator $A\in\mathcal{A}(\mathcal{O})$ on the vacuum $\left|0\right\rangle$, is based on the following relation

$$\begin{aligned} &A\left|0\right\rangle \rightarrow \{\left\langle p_1,p_2...p_n\left|A\right|0\right\rangle\}_{n\in\mathbb{N}} \\ &\left\langle 0\left|A^*\right|p_1p_2...p_n\right\rangle^{in} = {}^{out}\left\langle -p_n,-p_{n-1},...-p_{k+1}\left|A^*\right|p_1p_2...p_k\right\rangle^{in} \end{aligned} \tag{8}$$

where the second line is the *crossing identity* (last section). In other words, a "bang" on the vacuum leads to a state with an arbitrary high number of particles, or to phrase it in the vernacular manner of Murphy's law: *what can occur* (as the outcome of a bang subject to the superselection rule) *actually does occur.* To act on the vacuum more softly, so that (as in the case of partial charges) the excitation of states with arbitrary high particle number is suppressed, one has to resort to "quasilocal operators" [8]. The $-p$ refers to a well defined analytic continuation from the positive mass shell to the negative mass shell so that the last aspect of virtuality of (8) is removed by stating that the actual particle production in a bang on the vacuum is uniquely determined by the kind of "hammer" B used for banging[19]; or returning to more scientific parlance, the vacuum formfactor of a local operator B determines its associated general formfactor (with the same total number of particles) by analytic continuation.

The "bang on the vacuum" concept is less metaphoric than the picture of the QFT vacuum as a "broiling soup" of virtual pairs [41]. No conceptual headstand like "allowing the uncertainty relation for a short time to invalidate the energy conservation" is necessary. In fact the uncertainty relation is connected with the position operator which is not a well-defined (frame-independent) object in QFT; hence an uncertainty relation has no conceptual place in QFT since there is no position operator. Its covariant QFT counterpart is the increase of the localization entropy/energy with the sharpening of localization by compressing the fuzzy surface sheet of size ε in definitions of partial charges (14) and localization-entropy (see the next section).

It is quite interesting to add another fact about the local banging. Whereas the application of all global operators generates the full Hilbert space, the application of a local algebra $\mathcal{A}(\mathcal{O})$ does not, as one may naively think, generate a closed subspace as it does in QM; but rather generates a dense subspace of H (the Reeh-Schlieder theorem [8]) which changes with the localization region without loosing its property of being dense. This confirms that by "changing the hammer" within $\mathcal{O}$ one gets arbitrarily close to any particle state. Although this does not yet reveal the thermal aspects of localization, it does point to another property which shows that behind Haag's visualizing of local observable in

[18] Causally localized operators applied to the vacuum ("bangs") create states with the full energy spectrum; in the presence of interactions these states also contain the full in (or out) particle spectrum.

[19] Depending on its more or less sharp surface (fuzzy boundary) the vacuum polarization clouds are stronger of weaker.

terms of finite extension and duration of measurements loom quite metaphoric details if one insists to re-express its mathematical precision in terms of idealized manipulations on experimental hardware. The only understood case is that of a localization in a wedge (Unruh's accelerated observers). But as previously stated, metaphoric aspects of foundational concepts are perfectly acceptable as long as the mathematical consequences can be clearly formulated. In fact from a philosophical viewpoint one would even expect that this discrepancy between the intuitive content of principles and the precise reformulation of their mathematically rigorous setting increases as theories become more inclusive and fundamental. For the case of compact localization this process of reformulation in terms eludes a visualization in terms of physical hardware.

The great remaining problem which will decide the future of QFT is therefore to find nonperturbative techniques which are in accordance with the intrinsic holistic localization of QFT and are mathematically controlled. For this we first have to understand in more detail what this intrinsic nature of QFT consist of. The finding will enable us to decode the E-J conundrum (section 4).

3 Local quantum physics: modular localization and its thermal manifestation

Although by 1929 [42] Jordan knew through previous work with Pauli that QFT led to manifestations which were distinctively different from those of infinite degree of freedom QM, the idea that one needs a new conceptual setting did not yet take hold. From the viewpoint of the formalism of Lagrangian quantization there was no visible distinction, except that relativistic QFT was Poincaré covariant and, at least in the presence of interactions, did not allow a "first quantized" wave function formulation. Apart from occasional flare-ups which found their expression in sayings like: "putting QFT on its own feet", or " QT without classical (quantization) crutches" [42], there was as yet no concerted effort to understand both quantum theories in terms of different intrinsic principles rather than of a shared quantization formalism. The conceptual-mathematical setting of QM reached its closure already in the 30s in the work of John von Neumann and Hermann Weyl before it also entered textbooks. At that time the not even the vacuum polarization phenomenon of QFT was properly understood.

Foundational work on QFT started more than two decades later, partly because the fundamental differences were initially not perceived as such. Without this move the old ultraviolet infinities, which plagued QFT for more than a decade, would have continued to be a cause confusion. The number of renormalizable couplings of pointlike fields in d=1+3 is finite and all of them are known. The standard model, i.e. the joining of weak, electromagnetic and string interactions under the roof of gauge theories, was the last big push; after this, particle theory entered an already 40 years lasting period of conceptual stagnation.

The only idea which requires a different (not yet completely elaborated)

form of perturbation theory is the use of string-localized fields of higher spin [43][44] which formally improve their short distance behavior and enlarges the number of possible models which fulfill the power counting renormalizabilty criterion. For the s=1 gauge theories their subset of point-localized fields agrees with the gauge invariant fields of the quantization gauge approach. The use of string-localized potentials leads to fields in positive metric Fock-spaces with short distance dimension $d_{s.d} = 1$; this enlarges the potentially renormalizable couplings to infinitely many possibilities. Details about how modular localization leads to these new string-localized fields and what can be expected from their perturbative use can be found in [54] [39] [44] [38].

In Haag's approach a QFT is defined in terms of a net of local (von Neumann) operator algebras $\{\mathcal{A}(\mathcal{O})\}_{\mathcal{O}\subset M_4}$. Fields in the sense of Wightman are global objects $\Phi(x)$ which upon smearing with $\mathcal{O}$-supported test functions $supp f \subset \mathcal{O}$ become localized unbounded operators $\Phi(f)$ affiliated to local algebras $\mathcal{A}(\mathcal{O})$. The global nature of the field is reflected in the fact that it serves as a *generator for all local algebras*. One believes that all physically relevant nets of local algebras are generated by local fields, but the lack of a general proof is not very important because the algebraic setting has been shown to contain the full interpretation of the theory, in particular the important relation to particles in terms of scattering theory [8].

The algebras fulfill a set of obvious consistency properties which result from the action of Poincaré transformations, spacelike commutation relations and causal completeness properties. Instead of the Poincaré transformation law of covariant spinorial $\Psi^{(A,\dot{B})}$ fields, it is only required that the transformed operator $A \in \mathcal{A}(\mathcal{O})$ belongs to the operator algebra of the transformed region. The causality requirements are

$$\begin{aligned} &[A,B]=0,\ A\in\mathcal{A}(\mathcal{O}),\ B\in\mathcal{A}(\mathcal{O}')\subseteq\mathcal{A}(\mathcal{O})',\ \textit{Einstein causality} \\ &\mathcal{A}(\mathcal{O})=\mathcal{A}(\mathcal{O}''),\ \textit{causal shadow property},\ \textit{causal completion of}\ \mathcal{O} \end{aligned} \tag{9}$$

Here the first line the algebraic formulation of the statistical independence of spacelike separated events; the upper dash on the spacetime region denotes the spacelike disjoint region, whereas on the algebra it stands for the commutant algebra. The second line is the local version of the aforementioned time-slice property [45] where the double causal disjoint $\mathcal{O}''$ is the causal completion (shadow) of $\mathcal{O}$.

One of the oldest observations about peculiar consequences of causal localization in QFT is the Reeh-Schlieder property [8] i.e. the denseness of $\mathcal{A}(\mathcal{O})\left|0\right\rangle$ in H. Together with Einstein causality this denseness property leads immediately to the *standardness* of the pair $(\mathcal{A}(\mathcal{O}),\left|0\right\rangle)$ i.e. the property that $\mathcal{A}(\mathcal{O})$ acts cyclic (the density property) and (in contrast to the global algebra) separating (contains no annihilators) on $\left|0\right\rangle$.

This property attracted the attention of philosophical inclined particle physicists as no other quantum aspect. Its counterintuitive aspect, admitting to change the situation "behind the moon" by doing something in an arbitrary small laboratory during an arbitrary small duration, is in a way more radical

than even the quantum mechanical "Schroedinger cat" paradox and the EPR entanglement[20] phenomenon.

As mentioned before, in ("the second quantized" formulation of) QM localization is related to the spectral theory of a position operator and refers to the subspaces associated with the projectors of its spectral decomposition; the objects which are being localized in a spatial region $\mathcal{O}$ are subspaces $H(\mathcal{O}) \subset H$ or state vectors $\psi(f)\left|0\right\rangle$ in which the test functions have their support in the spectrum of the selfadjoint position operator $spec\mathbf{x} \subset R^3$. This state-localization, which is directly related to Born's probability concept, plays also a role for relativistic wave function. As Newton and Wigner showed [8], it depends on the frame of reference i.e. is not part of the Lorentz covariant observables of QFT. In the setting of "direct particle interaction" (DPI) [12] [13], a kind of relativistic QM with interaction, the covariance is only recovered in the scattering limit i.e. the resulting Möller operator and its S-matrix are the only covariant operators, in fact they are even invariant as they should be. Whereas DPI is primarily a quantum mechanical construction of a relativistic S-matrix, QFT is a theory of covariant localized observables and the invariance of the S-matrix is a direct consequence of the covariance of local observables. In QFT there are covariant fields but no particles at finite times; particles and fields harmonize only asymptotically; any attempt to enforce their coexistence at finite times in the presence of interactions is bound to fail (see below). In DPI particles exist at all times at the prize of absence of covariant observables besides the global S-matrix.

The Born localization is also important for QFT where its range of application is limited to wave functions. It turns out that the centers of wave packets which mark the region of largest Born probability density for large times follow linear orbits with sharply defined velocities which agree with what one expects from the result from sharp causal propagation i.e. in this asymptotic (effective) sense both localizations coalesce. All the alleged superluminal violations of causality which appeared and still appear in articles and journals have their origin in the incorrect identifications of the two localization concepts for finite times. The velocity of sound in QM or the limiting velocity c in DPI only attain their precise meaning (sharp value) at large times.

The mathematical backup of the QFT localization is the *modular operator theory* in the setting of operator algebras (for references consult [13]). Its most important operator is the unbounded involutive Tomita operator $S_{\mathcal{O}}$ (see below) whose domain $domS_{\mathcal{O}}$ is intimately related to the Reeh-Schlieder dense subspaces. A closer examination of the Tomita S-operators (next section) in case of $\mathcal{O} = W$ reveals that their domain is entirely determined by the representation of the Poincaré group, a property which is shared between the interacting and their associated free incoming operators. The interaction only enters in the specific way S_W maps vectors from $domS_W$ into their image in S_W.

As already mentioned, the conceptual consequences of this theorem exerted an enormous attraction to philosophers of science as can be seen from articles

[20] In quantum information theory the word "nonlocal" has a different meaning than in QFT.

under the heading "Reeh-Schlieder wins against Newton-Wigner" [46] which if phrased in the terminology of the present article would read "modular localization versus Newton-Wigner localization". This antagonistic title represents the "half glass empty view" of the situation; in the "half glass full view" of the present paper one would instead emphasize that the two localizations become compatible at the only place where it really matters, namely in the asymptotic scattering region [13].

In the presence of modular localization it is less important to understand the individual differences of operators within a local algebra, since the fact that they share the localization in a spacetime region $\mathcal{O}$ and that the state vectors they create from the vacuum have a nonvanishing inner product with all n-particle states already suffices to extract the S-matrix [8]. LQP derives all properties of particle theory from ensembles of operators and (apart from generating conserved currents which generate symmetries) sidesteps properties of individual operators. Higher precision in characterizing observables belonging to specific spacetime regions amount to improvement in localization, just as in real experiments where the precision in measuring localizations of charges, momenta, masses and spins depend on the improvement of the internal structure of counters and the geometric relation between sources and counters. As explained in the first section, the appropriate framework for formulating causality properties is that of nets of spacetime-indexed *von Neumann algebras*; only in such operator algebras can the weak closure referring to states be replaced by the double commutant operation i.e. $\mathcal{A} = \mathcal{A}''$ (which is a pure algebraic concept) and the causal completion matches the algebraic notion of commutant used in the formulation of Einstein causality. These operator algebras may be obtained from the more abstract C*-algebras by representation theory, a viewpoint which is particularly helpful in the presence of unitarily inequivalent representations. As previously mentioned the modular aspects of the operator algebraic setting entered particle physics for the first time in the conceptually correct formulation of the problem of *statistical mechanics of open systems* [47].

Already at the beginning of the 60 it was known that local algebras could not be of quantum mechanical type I_∞. At the time of the first LQP paper [45] after Haag's talk at the 1957 Lille conference [14], most people (including myself) still tacitly believed that the type should be I_∞; in part because almost nothing was known about type III algebras and the only kind of algebra one met in QM was that of all bounded operators $B(H)$ in a Hilbert space which is I_∞. Two years later Araki showed that it belongs to the type III family [8]. A decade later, after the refinement of the type classification by Connes, an important addition by Haagerup and the geometric physical identification of the modular objects of wedge-localized QFT operator algebras $\mathcal{A}(W)$ by Bisognano and Wichmann [21], all the concepts which link localization in QFT with thermal aspects and made the monad unique (it represents just one isomorphy class) were in place.

Around the same time there were independent observations about QFT in curved space time (CST) [8], more precisely restrictions to partial spacetime regions in front of black hole event horizons. It was noted that, unlike classical matter, the presence of quantum matter leads to Hawking radiation at the

Hawking temperature. A similar thermal manifestation was pointed out in the setting of a Gedankenexperiment by Unruh [25]. In order to localize an observer in such a way that his accessible spacetime region is a Rindler wedge W, he has to be uniformly accelerated in which case his world line is traced out by applying the wedge-related Lorentz boost to his start at t=0 inside the wedge. His proper time is different from the Minkowski time and depends on the constant acceleration. Taking this into consideration, the observer's Hamiltonian is only different from the dimensionless generator of the Lorentz boost by a dimensionfull numerical factor.

Even if the tiny associated temperature will never be measured and Unruh's proposal always remain a Gedankenexperiment, the consequences of modular localization for QFT are of pivotal structural importance for QFT. As will be seen they are not only helpful in order to completely resolve the Einstein-Jordan conundrum at the cradle of QFT, but even more important to understand the conceptual basis of subtle properties of particle theory as the particle crossing property of formfactors and the existence of nontrivial models and controlled ways to approximate them. At present there is no other setting than that of modular localization which has a chance to place QFT side by side to all the other already conceptually closed theories in the pantheon of theoretical physics.

The Unruh Gedankenexperiment is a special case of a more general setting of modular localization[21] [10] which describes the position of the dense subspace in terms of domains of the unbounded Tomita involution S. These domains are determined in terms of the unitary representation of the Poincaré group, but for knowing the action of the operator itself, one needs to know dynamic aspects of a QFT which leads to those localized states. This localization is intrinsic to the Wigner representation theory for positive energy representation. In the absence of interactions there is a direct functorial passage from subspaces of states to subalgebras. Since this has been explained in detail in [23][49][43][39] and since the operator algebra localization for interacting QFT is more important in the present context than that of subspaces, we refer the reader to the literature.

The general modular operator theory starts with the definition of the Tomita S-operator (section 1, 2 3) and the action of the operators Δ^{it}, J on the algebra $\mathcal{A}$ obtained from the polar decomposition of S. The prerequisite of "standardness" for the applicability of the T-T modular theory to localized algebras of QFT are always fulfilled thanks to the Reeh-Schlieder theorem [8]. It guaranties the universal validity of modular theory for local algebras of QFT with respect to any finite energy state as long as their causal closure is not the global algebra (which contains all operators and hence in particular annihilators of finite energy states). Unless specified otherwise, Ω in the sequel denotes the vacuum. In fact the $domS$ is nothing else than the closure of the Reeh-Schlieder subspace in the graph norm of S.

In QFT the Δ^{it} for the case of $\mathcal{O} = W$ is kinematic in the sense that it only depends on the representation of the Poincaré group $\Delta_W^{it} = U(\Lambda_W(-2\pi t))$.

[21] This was pointed out by Sewell [26] (but apparently not accepted by Unruh who maintained that the modular theory has nothing to do with the effect which bears his name).

Hence it is shared between all QFTs which live in the same Hilbert space and in addition of sharing the vacuum have the same particle content. On the other hand the modular reflection J and therefore S depend on the interaction through the S_{scat} matrix

$$J_W = J_{0,W} S_{scat} \tag{10}$$

where the subscript 0 refers to the interaction-free algebra generated by the incoming fields and S_{scat} is the scattering matrix. This follows from the work of Jost [31] on the TCP operation; is constructive use in for model constructions was noted in [10]. In this case of $\mathcal{O} = W$ the $\Delta^{i\tau}$ is (up to a scaling factor) the W-preserving Lorentz boost. In fact according to the B-W theorem [8] $\Delta_W^{i\tau} = e^{-i2\pi\tau K}$ with $K =$ infinitesimal generator of the W-preserving Lorentz boost, and $J = TCP$ up to a π-rotation around the z-axis.

The central result of the Tomita-Takesaki modular theory is the T-T theorem which states that $\Delta^{i\tau}$ defines a (modular) automorphism of the operator algebra and J an anti-isomorphism into its commutant

$$\Delta^{i\tau} \mathcal{A} \Delta^{-i\tau} = \mathcal{A},\ J\mathcal{A}J = \mathcal{A}' \tag{11}$$

Although the domain of the Tomita S-operator (except for wedge-localized algebras), allows no direct description in terms of the Poincaré group for subwedge regions, this information can be obtained from intersections of the real subspaces $\mathcal{K}$ associated with S_W

$$\mathcal{K}_W = \{\psi; S_W \psi = \psi_W\},\ \mathcal{K}_\mathcal{O} = \cap_{W \supset \mathcal{O}} \mathcal{K}_W \tag{12}$$

and hence even these domains are (indirectly) of kinematic origin. The dynamic content of subwedge reflections $J_\mathcal{O}$ is however not known.

The general modular localization situation is more abstract than its illustration in the context of the Unruh Gedankenexperiment since the generic modular Hamiltonian is not associated with any spacetime diffeomorphism; it rather describes a "fuzzy" movement which respects the causal boundaries (the horizon) but acts somewhat nonlocal on the inside bulk; very little is known about properties of modular Hamiltonians. But even in case where the analog of a localized Unruh observer is not available, the mere knowledge about the *existence* of a modular Hamiltonian is of great structural value, since it allows to give a mathematical precise quantum physical description of the locally restricted vacuum as an impure (singular KMS) state associated with the intrinsically determined modular Hamiltonian. The KMS property formulated in terms of the restriction of the vacuum to the algebra of the localization region reads

$$\begin{aligned} &\langle AB \rangle = \left\langle B e^{-H_{mod}} A \right\rangle,\quad \Delta = e^{-H_{mod}},\ A, B \in A(\mathcal{O}) \\ &\langle AB \rangle \neq \langle A \rangle \langle B \rangle \quad if\ \ [A, B] = 0\ \textit{in contrast to QM} \end{aligned} \tag{13}$$

Here the (unbounded) modular operator Δ and its associated modular Hamiltonian H_{mod} are associated to the "standard pair" $(\mathcal{A}(\mathcal{O}), \Omega)$. In the case of heat bath thermality (statistical mechanic) the Hamiltonian is the standard Hamiltonian which implements time translations in Minkowski spacetime, $\mathcal{O}$ is replaced

by the global spacetime $\mathbb{R}^4$ and the vacuum state Ω is now the GNS vector [35] of the thermal equilibrium state. The KMS property associated with the thermal manifestation of the vacuum setting of QFT serves primarily to direct attention to impure KMS nature of restricted vacua. The modular Hamiltonian changes if one enlarges the algebra by adding observables localized outside.

Since two localized operators $A, B \in \mathcal{A}(\mathcal{O})$ belong to a continuous set of algebras with larger localization, there is a continuous set of modular Hamiltonians which lead to KMS commutation relations with different Δ for the same localized pair of operators. This leads to a continuous infinity of KMS relation for a given pair and shows that QFT is a much tighter and more fundamental theory than QM. This rather subtle property of QFT illustrates again that, despite the immediate intuitive appeal of Haag's LQP setting, its mathematical consequences are anything but intuitive.

The thermal KMS property as a consequence of modular localization is primarily an attribute of an ensemble of observables which share the same localization region. As a consequence it is also a relation which each individual operator obeys, but the probability notion coming with thermal ensembles always reminds us that, unlike the situation in QM, the ensemble interpretation is intrinsic and does not have to be added. The important point here is that it would have been difficult to discover this infinite set of relations without knowing anything about the nature of modular localization within Haag's setting of LQP [8].

It is not conceivable that with the full knowledge about the thermal consequences of quantum causal localization Einstein would not have accepted this probability since he used probabilities of thermal ensembles in his fluctuation arguments for the existence of photons. QM is a global setting and its Born-localization and related probability based on the spectral decomposition of a position operator is ostensibly referring individual events. If the less fundamental QM could be derived from QFT in a limit which maintains the QFT probability but destroys its modular localization including its concomitant thermal manifestation one may even speculate that Einstein may have reconciled himself with Born's probabilistic interpretation of events in QM.

The old pre-renormalization perturbation theory failed precisely because quantum mechanical perturbation methods were used which do not keep proper track of covariance and causality. The covariant formulation at the end of the 40s consisted of recipes in which the role of the causal locality principle was not clearly visible so that problems as subvolume energy fluctuations (for which covariance was not of much help), lacked a precise conceptual understanding. In the case of the oscillator decomposition of Jordan's quantum field theoretic fluctuation model, the approximation of only occupying oscillator levels in a certain frequency range generally destroys the localization and thermal aspects required by QFT [5]. In an unguided oscillator approximation of the subvolume fluctuations one almost certainly destroys the holistic localization. The way to maintain it is the implementation of the LQP split property [8]; but since this refers to operator algebras, this leads to problems which are more difficult than the distribution-theoretical single operator calculation of "fuzzy" boundaries in

the calculation of partial charges in section 2.

It was noted elsewhere [30] that the holistic aspect of modular localization renders the use of quantum mechanical ideas of global level occupation (used e.g. in some estimates of cosmological vacuum energy density) potentially misleading[22].

Among the properties which cannot be ascribed to an individual operator but corresponds (similar to the E-J fluctuation) to a localized algebras is the problem of *localization-entropy* as a measure of localization-caused vacuum polarization. The entropy of sharp localization is infinite; unlike the infinities in the old ultraviolet catastrophe this is a genuine infinity in QFT; in fact in the absence of a position operator it represents the QFT analog of the QM uncertainty relation. There is a close connection with the infinite volume infinity of the heat bath entropy. For chiral theories on the lightray there is a rigorous derivation of the well-known linear increase (the "one-dimensional volume factor" L) of the heat bath entropy and the logarithmic growth of the QFT entropy with decreasing attenuation distance ε of vacuum polarization which are related[23] by -ln$\varepsilon \sim L$ as a result of a conformal isomorphism (next section).

In higher dimensions there are rather convincing arguments that the limiting behavior for $\varepsilon \to 0$ for the dimensionless entropy is the same as in the increase of the dimensionless partial charge (7). For the same geometric situation as in case of the partial charge (7) this suggests

$$V_{n-1}\left(kT\right)^{n-1}|_{T=T_{\mathrm{mod}}} \simeq \begin{cases} ln\left(\varepsilon^{-1}\right), & n=2 \\ \left(\frac{R}{\Delta R}\right)^{n-2}, & n>2 \end{cases} \tag{14}$$

where the volume proportional (dimensionless) entropy on the left hand side is the standard heat bath entropy; the n-2 power on the right hand side represent the n-2 transverse directions in n dimensions. Whereas, as a result of the existence of an inverse Unruh effect [24], the derivation of the n=2 localization is a consequence of that isomorphism, the n>2 case is supported by the analogy to the partial charge. It also agrees with 't Hooft area behavior from the brickwall assumption [34] which is matched with Bekenstein's classical area law by using for the free parameter ε the numerical value of the Planck length.

Since a rigorous implementation of the split formalism is still missing, there is room for another idea which amounts to a multiplicative logarithmic modification for n>2 by $ln\frac{R}{\Delta R}$. In that case the box-volume of the heat bath side would correspond to a volume of a box for which two sides are transverse and one (which accounts for the ln factor) would be lightlike, as in the chiral case n=2. This could be seen as a weak version of an *inverse* Unruh situation: there is no isomorphism between the two systems but there still remains a close analogy between heat bath and localization-caused thermal behavior [40]. From a

[22] It is interesting that Ehlers (see [5]) mentions the problem of cosmological constant in connection with the Einstein-Jordan conundrum, thus suggesting that in both problems the role of the vacuum polarization has not been properly understood.

[23] In a more detailed description of chiral theories the fuzzyness ε can be expressed in terms of a conformal invariant ratio of 4 points which are the end points of a smaller interval included in a bigger one [40].

mathematical viewpoint this suggests that the monad in its role of describing a thermodynamic limit of heat bath system and that used for localization are mainly different in terms of their different physical parametrization.

The strongest illustration of the holistic aspects of QFT as compared to QM is the characterization of models of QFT (including the quantum matter content as well as its ordering spacetime symmetry structure) in terms of modular positioning which was already mentioned in the introduction. This was first observed in chiral QFT on the lightray, permit a characterization in which instead of spacetime (interval) ordered quantum matter (nets of operator algebras) permitted a complete characterization in terms of the relative positioning of a finite number of monads. After preliminary observation on "quarter circle inclusions" [50], the appropriate mathematical setting was found in terms of the notion of *halfsided modular inclusions [51], i.e.* inclusions of one monad in another which share their standard vector and for which the modular group Δ^{it} of the bigger monad compresses the smaller one for one sign of t. It turns out that the monad requirement on the operator algebras can be omitted; it follows from the halfsided modular property of the inclusions; the Möbius symmetry together with the one-dimensional spacetime on which it acts are encoded into the modular inclusion of two operator algebras in a Hilbert space. A comprehensive discussion of the relation of *strongly additive* nets via modular inclusions to Möbius covariant nets can be found in [52].

This abstract algebraization of the concepts behind the standard description QFT in terms of geometrically ordered quantum matter has a generalization to higher dimensions [28][13]. The number of monads is still finite and increases with spacetime dimensions of the QFT which one wants to construct. Since the monad has no internal structure all the physical and mathematical richness of QFT comes from relative modular positioning of copies of a monad in a shared Hilbert. Although in the present state of modular operator theory QFT models (with the exception of chiral theories) cannot be classified and constructed in this way, it illustrates an important aspect of the holistic nature of QFT and the unexplored power of its modular localization principle.

4 The d=1+1 Jordan model and the isomorphism which solves the Einstein-Jordan conundrum

With the *locally restricted vacuum* representing a highly (non-tracial) impure state with respect to *all* modular Hamiltonians $H_{mod}(\check{\mathcal{O}})$, $\check{\mathcal{O}} \supseteq \mathcal{O}$ on local observables $A \in \mathcal{A}(\mathcal{O}) \subset \mathcal{A}(\mathcal{O}'')$, a fundamental conceptual difference of QFT and QM has been identified. In this section it will be shown how modular localization solves the E-J conundrum in terms of an operator-algebraic isomorphism between Jordan's model and its Einstein statistical mechanics analog. In view of the importance of modular localization in the present QFT research, this is much more than a historical retrospection; it is a precursor of the Unruh-

Hawking observation of an analogy between localization-caused and heat bath thermal behavior in QFT in a case where the thermal aspect is not only an analogy but where the two systems (after re-parametrization) are identical. Whereas modular localization has helped to view Unruh-Hawking situations as special cases of the defining structural property of causal localization in QT, the full solution of the E-J conundrum at the time of the historical dispute might have brought an aspect of QFT into the open (just a nice dream) which may have changed the path of history of QFT and in particular reconciled Einstein with the intrinsic ensemble probability of QFT.

In modern terminology Jordan's model QFT of a two-dimensional photon[24] is really a model of a chiral current. As a two-dimensional zero mass field which solves the wave equation it can be decomposed into its two u, v lightray components (omitting the u, v vector indices)

$$
\begin{aligned}
&\partial_\mu \partial^\mu \Phi(t,x) = 0,\ \Phi(t,x) = V(u) + V(v),\ u = t + x,\ v = t - x \qquad (15) \\
&j(u) = \partial_u V(u),\ j(v) = \partial_v V(v),\ \langle j(u) j(u') \rangle \sim \frac{-1}{(u - u' - i\varepsilon)^2} \\
&T(u) =: j^2(u) :,\ T(v) =: j^2(v) :, \quad [j(u), j(v)] = 0
\end{aligned}
$$

The model belongs to the family of conformal chiral models. The scale dimension of the chiral current is $d_{sd}(j) = 1$, whereas the energy-momentum tensor (the Wick-square of j) has $d_{sd}(T) = 2$; the u and v world are completely independent and it suffices to consider the fluctuation problem for one chiral component. The logarithmic divergence of the zero dimensional chiral $d_{sd}(V) = 0$ current potential V arises from the semiinfinite string-localization of V; a better behaved Möbius-group covariant fields is the charge-carrying sigma model field formally written as $expi\alpha V$ [53]. Since our argument only uses the Möbius covariance and localization of the Weyl algebra generated by the chiral current j, such details concerning the charge superselection structure and charge-carrying fields are irrelevant.

The E-J fluctuation problem can be formulated in terms of j (charge fluctuations) or T (energy fluctuations). It is useful to recall that vacuum expectations of chiral operators are invariant under the fractionally acting 3-parametric acting Möbius group (x stands for u, v), which for j reads

$$
\begin{aligned}
&U(a) j(x) U(a)^* = j(x + a),\ U(\lambda) j(x) U(\lambda)^* = \lambda j(\lambda x) \quad dilation \qquad (16) \\
&U(\alpha) j(x) U(\alpha)^* = \frac{1}{(-sin\alpha + cos\alpha)^2} j\left(\frac{cos\alpha x + sin\alpha x}{-sin\alpha x + cos\alpha x}\right) \ rotation
\end{aligned}
$$

[24] This terminology was quite common in the early days of field quantization before it was understood that that in contrast to QM, the physical properties of QFT depend in an essential way on the spacetime dimension. Jordan's d=1+1 "photons" and his later "neutrinos" (in his "neutrino theory of light" [55]) are not two-dimensional versions of 4-dimensional objects in the sense that in QM an oscillator chain always remains the same object independent of the dimension of the embedding space.

The next step consists in identifying the KMS property of the locally restricted vacuum with that of a global system in a thermodynamic limit state. For obvious reasons it is referred to as the *inverse Unruh effect,* i.e. finding a heat bath thermal system which corresponds to the restriction of the vacuum to the j-associated Weyl algebra $\mathcal{A}((a,b))$ localized on an interval.

Theorem 1 *([24]) The global chiral operator algebra $\mathcal{A}(\mathbb{R})$ associated with the heat bath representation at temperature $\beta = 2\pi$ is isomorphic to the vacuum representation restricted to the half-line chiral algebra such that*

$$(\mathcal{A}(\mathbb{R}), \Omega_{2\pi}) \cong (\mathcal{A}(\mathbb{R}_+), \Omega_{vac}) \tag{17}$$
$$(\mathcal{A}(\mathbb{R})', \Omega_{2\pi}) \cong (\mathcal{A}(\mathbb{R}_-), \Omega_{vac})$$

The isomorphism intertwines the translations of $\mathbb{R}$ with the dilations of $\mathbb{R}_+$, such that the isomorphism extends to the local algebras:

$$(\mathcal{A}((a,b)), \Omega_{2\pi}) \cong (\mathcal{A}((e^{-2\pi a}, e^{2\pi b})), \Omega_{vac}) \tag{18}$$

This can be shown by modular theory. The proof extends prior work by Borchers and Yngvason [56]. Let $\mathcal{A}$ denote the C^* algebra associated to the chiral current j[25]. Consider a thermal state ω at the (for convenience) Hawking temperature 2π associated with the translation on the line. Let $\mathcal{M}$ be the operator algebra obtained by the GNS representation and $\Omega_{2\pi}$ the state vector associated to ω. We denote by $\mathcal{N}$ the halfspace algebra of $\mathcal{M}$ and by $\mathcal{N}' \cap \mathcal{M}$ the relative commutant of $\mathcal{N}$ *in* $\mathcal{M}$. The main point is now that one can show that the modular groups of $\mathcal{M}$, $\mathcal{N}$ and $\mathcal{N}' \cap \mathcal{M}$ } generate a "hidden" positive energy representation of the Möbius group $SL(2,R)/Z_2$ where "hidden" [24] means that the actions have no geometric interpretation on the thermal net. The positive energy representation acts on a hidden vacuum representation for which the thermal state is now the vacuum state Ω.The relation of the previous three thermal algebras to their vacuum counterpart is as follows:

$$\mathcal{N} = \mathcal{A}(1,\infty),\ \mathcal{N}' \cap \mathcal{M} = \mathcal{A}(0,1),\ \mathcal{M} = \mathcal{A}(0,\infty) \tag{19}$$
$$\mathcal{M}' = \mathcal{A}(-\infty,0),\ \mathcal{A}(-\infty,\infty) = \mathcal{M} \vee \mathcal{M}'$$
$$\mathcal{M}(a,b) = \mathcal{A}(e^{-2\pi a}, e^{2\pi b}) \tag{20}$$

Here $\mathcal{M}'$ is the "thermal shadow world" which is hidden in the standard Gibbs state formalism but makes its explicit appearance in the so called *thermo-field* setting i.e. the result of the GNS description in which Gibbs states described by density matrices or the KMS stated resulting from their thermodynamic limits are described in a vector formalism. The last line expresses that the interval algebras are exponentially related.

In the theorem we used the more expicit notation

$$\mathcal{M}(a,b) = (\mathcal{A}(a,b), \Omega_{th}) = (\mathcal{A}(e^{-2\pi a}, e^{2\pi b}), \Omega_{vac})$$

[25] One can either obtain the bounded operator algebras from the spectral decomposition of the smeared free fields $j(f)$ or from a Weyl algebra construction.

Moreover we see, that there is also a natural space-time structure on the shadow world i.e. on the thermal commutant to the quasilocal algebra on which this hidden symmetry naturally acts. Expressing this observation a more vernacular way, one may say: *the thermal shadow world has been converted into virgin living space beyond the horizon of a localized Unruh world* [24]. In conclusion, we have encountered a rich hidden symmetry lying underneath the tip of an iceberg, of which the tip was first seen by Borchers and Yngvason [56].

Although we have assumed the temperature to have the Hawking value $\beta = 2\pi$, the reader convinces himself that the derivation may easily be generalized to arbitrary positive β as in the Borchers-Yngvason work. A more detailed exposition of these arguments is contained in a paper *Looking beyond the Thermal Horizon: Hidden Symmetries in Chiral Models* [24].

In this way an interval of length L (one-dimensional box) passes to the size of the split distance ε which plays the role of Heisenberg's vacuum polarization cloud $\varepsilon \sim e^{-L}$. Equating the thermodynamic $L \to \infty$ with the the limit of a fuzzy localization converging against a sharp localization on the vacuum side in $(e^{-2\pi L},\, e^{2\pi L})$ for $L \to \infty$ with the fuzzynes $e^{-2\pi L} \equiv \varepsilon \to 0,$ the thermodynamic limit of the thermal entropy passes to that of the localization entropy in the limit of vanishing ε

$$LkT\mid_{kT=2\pi} \simeq -\ln\varepsilon \tag{21}$$

where the left hand side is proportional to the (dimensionless) heat bath entropy and the right hand side is proportional to the localization entropy.

Although it is unlikely that a localization-caused thermal system is generally isomorphic to a heat bath thermal situation in higher dimensions (the strong inverse Unruh effect), there may exist a "weak" inverse Unruh situation in which the volume factor corresponds to a logarithmically modified dimensionless area law (previous section) i.e. $(\frac{R}{\Delta R})^{n-2}\ln(\frac{R}{\Delta R})$ instead of $(\frac{R}{\Delta R})^{n-2}$ where R is the radius of a double cone with a fuzzy surface and $\frac{\Delta R}{R}$ the dimensionless measure of the fuzzy surface. The box on the localization side (14) has two transverse- and one lightlike- extension and is the counterpart of the spatial box in a weak inverse Unruh picture. This would be different by a logarithmic factor from the area law which is suggested by the analogy to the behavior of vacuum polarization of a partial charge in the sharp localization limit (Section 2) and which also appears in 't Hooft's "brickwall" proposal [34] to make the derivation of the Hawking radiation of quantum matter in CST consistent with Bekenstein's classical area law. The present state of computational control of the split property is not able to decide between these two possibilities.

The above isomorphism shows that Jordan's imagined situation of quantum fluctuations interpreted as fluctuations in a small subinterval of a chiral QFT restricted to an interval which is Möbius equivalent to a halfline and therefore isomorphic to Einstein's thermodynamic limit system on the full line. Although the thermal aspect of a restricted vacuum in QFT is a structural consequence of causal localization, the general identification of the dimensionless modular temperature with a temperature of a heat bath system (the inverse Unruh effect), or, which is equivalent, the modular "time" with the physical time, is an

unsolved conceptual-mathematical problem [18].

5 The role of modular localization in the ongoing research

The use of the Einstein-Jordan conundrum as an indicator of the presence of radically different aspects of QFT would remain in the philosophic-historical realm if the LQP algebraic viewpoint would not also lead to new results beyond Lagrangian quantization and perturbation theory. In fact these new local algebraic methods led, for the first time in the history of QFT, to an existence proof in the presence of interactions in a family of d=1+1 models with realistic (non-canonical) short distance behavior[26]. This is the content of pathbreaking work by Gandalf Lechner [58][72]. In this way the the existence of certain integrable models about which there were already exact computational results for certain formfactors [59] was finally secured and a new direction for future more general attempts was pointed out. What is most surprising is the radically new method of construction. Whereas the Lagrangian (or functional integration) quantization starts from a classical action and the result of the perturbative calculation reveals its quantum interpretation ("bottom-to-top"), in the LQP approach to QFT one starts from foundational principles and tries to find the appropriate mathematical concepts to implement them (top-to-bottom).

Bottom-to-top methods aim typically at perturbative series for correlation functions of fields ("off-shell"); particle aspects ("on-shell") as scattering amplitudes and formfactors usually appear at a later stage. Despite all its merits concerning observational agreements, the well-known divergence of perturbative series limits its use; mathematically-controlled approximations are not known, let alone proofs of existence of interacting models. Since these problems are endemic, they could have their explanation in the direct use of rather singular objects (operator-valued distributions). Top-to-bottom approaches typically start from on-shell quantities as the S_{scat}-matrix or formfactors (interacting operators between outgoing bra and incoming ket states) whereas off-shell correlation functions only appear in later stages. The important observation is that in QFTs with a complete particle interpretation the S_{scat}-matrix is a relative modular invariant of the wedge-localized algebra (10). This suggests that the position of a wedge algebra in an interating theory is determined in terms of the S_{scat}-matrix and the corresponding free field wedge algebra.

This idea can be explicitly tested for a particular class of d=1+1 theories whose S_{scat} is determined in terms of multi-component two-particle scattering functions fulfilling the Yang-Baxter relation. It has been known for a long time that elastic scattering solutions of the so-called S-matrix bootstrap setting (Poincaré invariance, unitarity and crossing) cannot exist in higher dimensions; the necessary presence of inelastic processes prevents the constructive use of

[26] The previous existence proofs were limited to models with a canonical (superrenormalizable) short distance behavior [61].

such general properties. Elastic S_{scat}-matrices in d=1+1 are however susceptible to a classification within a bootstrap setting. Such a classification in terms of symmetries leads to families of models which only in a few cases make perturbative contact with Lagrangians. The second step, namely the construction of formfactors of a would-be QFT associated with these scattering functions known as the "bootstrap-formfactor project", has led to a wealth of results [60].

QFT is the only area of theoretical physics which, apart from certain d=1+1 models with canonical (superrenormalizable) short distance behavior [61], remained in its almost 90 years history without mathematical support concerning the existence of interesting interacting models. The "factorizing models", with their nontrivial renormalizable short distance behavior and their integrable formfactors associated with elastic S-matrices, present a fascinating "theoretical laboratory" for the study of this ultimate conceptual challenge, namely to secure the mathematical existence and (in case of non-integrable models) provide mathematically controlled approximations.

This last step in this construction is the verification that the computed formfactors really fulfill all the properties which entered in form of an Ansatz into their construction. This is the most interesting and important part of the construction since it includes the new LQP idea of proving existence of a family of models with non-canonical short distance behavior [58]. Unlike the Lagrangian quantization setting, ultraviolet and renormalization aspects play no role in this construction. In a few cases a perturbative comparison of formfactors with those obtained from Lagrangian quantization identified them with Lagrangian models whose integrability was based on quasiclassical arguments (Sine-Gordon, Sinh-Gordon,..); but for most of the integrable model it was necessary to baptize them with names referring to symmetries or analogies with lattice models.

The first observations about a relation between these "bootstrap-formfactor methods" with localized operator algebras and their generators go back the 90s [10] and consisted in attributing a spacetime interpretation to the Zamolodchikov-Faddeev algebra generators which the Zamolodchikov brothers introduced as a simplifying algebraic device for the classification of factorizing S_{scat}-matrices. It turned out that the Fourier transforms of these generalized creation/annihilation operators are generators of wedge-localized algebras. The existence proof in d=1+1 requires an even more subtle step: to show that a nontrivial double cone localized algebra can be obtained from intersecting a wedge-localized algebra $\mathcal{A}(W)$ with a translate of its opposite $\mathcal{A}(W')$. This was achieved by the use of modular nuclearity in a pathbreaking work by Lechner [58]. These ideas have been extended by deformation theory (deformation of free fields for models without bound states [72]), and meanwhile integrable models which even by experts are considered to be difficult (as the $O(N)$-model [63]) are in the range of the modular nuclearity arguments [64] which already secured the existence of simpler models.

In contrast to Wightman's setting of QFT in terms of correlation functions of fields, Haag's formulation of a QFT in terms of local nets of operator algebras fortunately (since this seems to be prohibitively difficult) does not require an explicit construction of singular fields (operator-valued distributions) since all

observationally important quantities (S-matrix, formfactors) can be computed in the LQP setting. Factorizing models and chiral models [62] are presently the only QFTs for which the operator-algebra based methods led to existence proofs. These are examples par excelence for what is meant by "top-to-bottom calculations".

The algebraic method reveals much more than a new construction scheme of certain integrable models. It also leads to a foundational understanding of the crossing property and an intrinsic distinction between integrable and nonintegrable (the typical case) based on properties of generators of wedge-localized operator algebras. The important observation is that one-particle vacuum-polarization-free-generators (PFG) of wedge-localized algebras and their generalized multi-particle "emulats" come with two distinctively different operator properties [65]: either their domains are translational invariant and they permit a Fourier-transformation or their domains are only invariant under those transformations which leave the wedge invariant. In the first case one can show that either $S_{scat}=1$ or (and this requires d=1+1) S_{scat} is elastic[27], whereas in the second case one has to cope with the full S_{scat} which couples the two-particle state to all higher particle states within the same superselection sector.

This situation permits a very elegant and useful definition of integrability in QFT which bypasses the (in QFT unhandy) definition in terms of infinitely many conserved charges in involution.

Definition 2 *([38]) The dichotomy integrable/nonintegrable in QFT is defined in terms of temperate/nontemperate PFG generators of wedge-localized algebras. Temperate generators with $S_{scat}\neq 1$ only exist in d=1+1*

It is very informative to present the definition of these two types of operators. In both cases they are bijectively related to wedge-localized incoming fields which share with the interacting algebras the same representation of the Poincaré group. Since therefore the modular unitaries Δ^{it} (the W-preserving Lorentz boost) for all wedge algebras with the same $\mathcal{P}$-representration coalesce, the domains $domS$ of the different Tomita S-operators agree and hence all states $|\eta\rangle$ in $domS$ permit a representation $A|0\rangle$ in terms of different operators, each uniquely affiliated with one of the different algebras. Our interest is to relate a specific interacting wedge-localized algebra $\mathcal{A}(W)$ with the corresponding interaction-free incoming algebra $\mathcal{A}_{in}(W)$.

Using the notation $A(f)$ for a free field smeared with a test function f with $suppf\in W$ and denoting the bijective related operator affiliated to $\mathcal{A}(W)$ by $(A(f))_{\mathcal{A}(W)}$ we have

$$\begin{aligned} &A(f)\left|0\right\rangle =A(f)_{\mathcal{A}(W)}\left|0\right\rangle ,\ A(f)=A(\check{f}) \\ &\mathcal{A}_{in}(W)\ni A\longleftrightarrow A_{\mathcal{A}(W)}\in\mathcal{A}(W) \\ &(A_{\mathcal{A}(W)})^{*}\left|0\right\rangle =SA_{\mathcal{A}(W)}\left|0\right\rangle =S_{scat}A^{*}\left|0\right\rangle \end{aligned} \tag{22}$$

[27]In fact it can be shown that there are no higher connected elastic S-matrices than $S_{scat}^{(2)}$ so that the n-particle amplitude is a combinatorial product of two-particle amplitudes.

where the $\in$ for unbounded operators means "affiliated with". W-smeared free fields $A(f)$ may also be written in terms of their momentum space creation/annihilation operators integrated with on-shell wave functions $\check{f}(p)$, which are the mass-shell projection of the Fourier-transformed test functions f. The $suppf \subset W$ property implies that $\check{f}$ is a boundary value of a function which is analytic in the $(0, i\pi)$ strip of the W-associated rapidity variable θ, where the lower boundary corresponds to the physical one-particle wave function $\check{f}$ and the upper boundary value is its complex conjugate (the c. c. of the anti-particle in case of complex fields). The third line in (22) states that the bijection does not preserve the passing to the adjoint and at the same time introduces the S_{scat}-matrix of the specific interacting model.

Only in the temperate (integrable) case the Fourier transformed $\left(\tilde{A}(p)\right)_{\mathcal{A}(W)}$ exist [65]. In this case the PFG behave very much like Wightman fields i.e. the $\left(A(\check{f})\right)_{\mathcal{A}(W)}$ exist for all wave functions (equivalently for all Schwartz smearing function). The only difference is its localization; it is in a certain restricted sense a nonlocal object: for $suppf \supset W$ it is fully nonlocal and for $suppf \subset W$ it remains wedge-localized independent of whether one sharpens this localization to $suppf \subset \mathcal{O} \subset W$.

It turns out that d=1+1 temperate one-particle PFG fulfill the Z-F algebra commutation relations which for the simplest case of a meromorphic scattering function $S(\theta)$ (no Yang-Baxter structure) reads:

$$\left(\tilde{A}(\theta)\right)_{\mathcal{A}(W)} \equiv Z^*(\theta),\ Z^*(\theta - i\pi) := Z(\theta) \tag{23}$$

$$Z(\theta)Z(\theta') = \delta(\theta - \theta' + i\pi) + S(\theta - \theta')Z(\theta')Z(\theta),\ S(-\theta) = \overline{S(\theta)} = S(\theta + i\pi)$$

where the definition in the first line is just a notation which permits to write the various commutation relations between creation/annihilation components in terms of one formula (second line) so that the δ-function only contributes to the mixed Z-Z^* commutation relations. The physical scattering range in $S(\theta)$ is $\theta > 0$ and the relations for the meromorphic function $S(\theta)$ in the second line represent unitarity and crossing.

For later purposes it is necessary to generalize the one-particle PFGs in terms of multiparticle "emulats". With $A(f_1,..f_n) \equiv: A(f_1)..A(f_n):$, $suppf_i \subset W$ the same modular arguments based on the domain properties of the Tomita S [65] which led to the wedge localized PFGs also apply to the emulats

$$A(f_1,..f_n)_{\mathcal{A}(W)}\left|0\right\rangle = A(f_1,..f_n)\left|0\right\rangle = \left|\check{f}_1,..\check{f}_n\right\rangle_{in}$$

As in the single particle case the unique affiliation of a multiparticle emulate with the interacting algebra $\mathcal{A}(W)$ is secured by its appropriately defined commutation with the commutant $\mathcal{A}(W)'$. The Wick-ordering simplifies the connection with n-particle states. Even for temperate emulats the bijection does not respect the algebraic multiplication structure i.e. $(A(f)A(g))_{\mathcal{A}(W)} \neq A(f)_{\mathcal{A}(W)}A(g)_{\mathcal{A}(W)}$.

For the derivation of the crossing identity for formfactors one needs the idea of *analytic ordering changes* of W-associated rapidities. Consider the formfactor

which describes the vacuum-polarization components of a local excitation in d=1+1. The n-particle components of a bra state $B^* |0\rangle$ is a symmetric function in the rapidities. The degeneracy of statistics may be encoded into an ordering prescription; assuming bosonic statistics we write

$$\langle 0 |B| \theta_1 ....\theta_n\rangle , \ B \in \mathcal{A}(W), \quad if \ \theta_1 > \theta_2 > .. > \theta_n \tag{24}$$

The ordering referes to the numerical values of the θ and not to the ordering of there indices. Other orderings are interpreted as the result of an analytic change of the θs. For theories with meromorphic scattering functions $S(\theta)$ the rapidity is a uniformization variable for the formfactors, i.e. the property of being meromorphic is passed to the formfactors. In this case an analytic transposition of two adjacent θ is given by the multiplication of the ordered formfactor by the scattering functions $S(\theta_i - \theta_{i+1})$ and repeated transpositions generate a non-degenerate "analytic" representation of the permutation group [66] and the Z-F algebra restricted to creation operators is an algebraic encoding of these analytic transpositions.

The crossing identity will be shown to result from the cyclic KMS identity which is a consequence of the modular localization property of $\mathcal{A}(W)$. We need it in the form

$$\left\langle 0|B \left(A^{(1)}\right)_{\mathcal{A}(W)} \left(A^{(2)}\right)_{\mathcal{A}(W)} |0\right\rangle = \left\langle 0| \left(A^{(2)}\right)_{\mathcal{A}(W)} \Delta B \left(A^{(1)}\right)_{\mathcal{A}(W)} |0\right\rangle \tag{25}$$

$$A^{(1)} \equiv: A(f_1, ..f_k) :, \ A^{(2)} \equiv: A(f_{k+1}, ..f_n) :, \ suppf_i \in W, \ B \in \mathcal{A}(W)$$

where two of the $\mathcal{A}(W)$ operators are emulates. Whenever an emulat acts on the vacuum it creates a W-localized multi-particle state ($\left|\hat{f}^{(a)}\right\rangle \equiv A(f)^* |0\rangle$):

$$\left\langle 0|B \left(A(\check{f}_1, ..\check{f}_k)\right)_{\mathcal{A}(W)} |\hat{f}_{k+1}, ..\hat{f}_n\right\rangle_{in} = {}_{out}\left\langle \hat{f}_{k+1}^{(a)}, ..\hat{f}_n^{(a)}|\Delta B|\hat{f}_1, ..\hat{f}_k\right\rangle_{in} \equiv \tag{26}$$

$$\equiv \int d\theta_1 .. \int d\theta_n \ \hat{f}_1(\theta), ..\hat{f}_n(\theta)_{out} \left\langle \bar{\theta}_{k+1}, .\bar{\theta}_n|\Delta^{\frac{1}{2}} B|\theta_1, .\theta_k\right\rangle_{in}$$

The bra states on the right side refer to antiparticles and the second line results from analytical continuation by $-i\pi$ of the complex conjugate antiparticle wave functions which are equal to the original wave functions, so that on both sides of the identity in the first line (26) the dense set of wave functions agrees. In order to write the left hand side in terms of an n-particle-vacuum formfactor, we need to know how the emulat act on a k-particle state. If it would be possible to extend this relation from the dense set in the space of $L^2(\theta)$-integrable wave function to wave functions with compact support in θ our analytic ordering assumption suggests the identification

$$\left\langle 0|B \left(A(\check{f}_1, ..\check{f}_k)\right)_{\mathcal{A}(W)} |\check{f}_{k+1}, ..\check{f}_n\right\rangle_{in} = \langle 0|B|\check{f}_1, ...\check{f}_n\rangle_{in} \tag{27}$$

$$for \quad supp\check{f}_1 > ... > supp\check{f}_n$$

whatever is the result of analytic reorderings may be. In fact since the statistics of the states and the symmetry inside a Wickproduct always permit to order inside the state and inside the emulat, it is only the relative ordering of the emulat cluster with respect to the particle cluster in the state which matters. This results in the particle crossing relation

$$\begin{aligned} \langle 0|B|\theta_1,..\theta_n\rangle_{in} &= {}_{out}\left\langle \theta_{k+1},..\theta_n|\Delta^{\frac{1}{2}}B|\theta_1,..\theta_k\right\rangle_{in} \equiv \\ &\equiv {}_{out}\left\langle \theta_{k+1}+i\pi,..\theta_n+i\pi|B|\theta_1,..\theta_k\right\rangle_{in}, \quad (\theta_1,..\theta_k) > (\theta_{k+1},..\theta_n) \end{aligned} \tag{28}$$

In other words the crossing identity is an extended form of the KMS identity; whereas the particle wave functions in the KMS relation are analytic (coming from W-localzed test functions), the crossing identity is a formfactor identity which only requires a relative ordering between bra and ket clusters but no integration with analytic wave functions. Rewritten interms of p-variables one recovers the standard form

$$\langle 0|B|p_1,..p_n\rangle_{in} = {}_{out}\langle -p_{k+1},..-p_n|B|p_1,..p_k\rangle_{in} \tag{29}$$

Since temperate PFG are synonymous with integrability, the assumptions made about analytic ordering changes and their relation to the action of emulats on particle states are established ex post facto using the exactly computed formfactors.

The conceptual and the calculational situation gets much more complicated in case of non-integrable QFT. Assuming again that the action of an emulat on a mutiparticle state can be related to an analytic change from an ordered situation, it is clear that this cannot be reduced to subsequent analytic transpositions. In other words the reordering in the second line

$$\langle 0|\, B(\tilde{A}(\theta))_{\mathcal{A}(W)}\, |\theta_1,..\theta_n\rangle = \begin{cases} \langle 0|\, B\, |\theta,\theta_1,..\theta_n\rangle\,,\ \theta > \theta_1 > .. > \theta_n \\ \sum_{|\ddddot{\vartheta}|} \int d\ddddot{\vartheta}\, F(\ddddot{\vartheta};\theta_1,.\theta_k;\theta)\, \langle 0|\, B\, |\ddddot{\vartheta},\theta,\theta_{k+1},..\theta_n\rangle \end{cases} \tag{30}$$

which is necessary if ... $> \theta_k > \theta > \theta_{k+1} >$ ..has to be done in one sweep. A single term in the resulting expression is an integral of a multiparticle state of total particle number $|\ddddot{\vartheta}| + 1 + n - k$ integrated with a function F over the multivariable configuration $\ddddot{\vartheta} \equiv (\vartheta_1,..\vartheta_m)$ of m=$|\ddddot{\vartheta}|$ particles. For the functions F there exists an Ansatz in terms of a "grazing shot amplitude" which can be written in terms of the full scattering amplitudes [38]. It expresses the fact that there is no direct interaction between the θ_i i.e. the θ must pass through the $(\theta_1,..\theta_k)$ cluster for activating an interaction. Hence the result of re-establishing the total ordering including θ leaves the configuration with smaller θ_i-values than θ unchanged, but cause a particle-number non-preserving change of the swept cluster.

From the exact results in [65] one knows that $(\tilde{A}(\theta))_{\mathcal{A}(W)}$ cannot make sense as operators. The tacit assumption underlying the above Ansatz is that such objects exist as bilinear forms between multiparticle states (formfactors of $(\tilde{A}(\theta))_{\mathcal{A}(W)}$) and that the action of operators $A(\check{f})$ on localized multiparticle domains can subsequently be constructed from the knowledge of the bilinear forms.

The generalization from one-particle PFGs to multiparticle emulats appears to require a clever notation more than new concepts.

The best way to place this attempt to get a hold on interacting QFT with a complete particle interpretation into a historical context is to view it as an extension of Wigner's representation theoretical particle setting (which by the use of modular wave function localization passes functorially to interaction free local operator algebras (section 3)) to the realm of interactions when the second quantization functor has to be replaced by emulation [38].

Explicit formulas for bilinear forms $(\tilde{A}(\theta))_{\mathcal{A}(W)}$ can be found in [38]. Although they pass the consistency check of reducing to the corresponding much simpler action of temperate PFGs, what remains to be done is to show that one can obtain operators $(A_{in}(f))_{\mathcal{A}(W)}$ with the claimed domain properties and, last not least, that these operators are wedge dual in the sense

$$\left\langle \psi \left| \left[JA(\hat{f})_{\mathcal{A}(W)}J, A(\hat{g})_{\mathcal{A}(W)} \right] \right| \varphi \right\rangle = 0, \;\; J = S_{scat}J_{in} \tag{31}$$

which is the wedge duality expressed in terms of the emulats. These problems are very difficult for non-integrable models, so that one does not expect an (positive or negative) answer in the near future.

The derivation of the crossing identity in the non-integrable case follows the same line of reasoning as for integrable models. Although the vacuum formfactors for non-integrable models are not meromorphic functions, it is plausible that their singularities are accounted for by the multiparticle scattering threshold (cuts from multiple roots) which the θ-uniformization cannot remove. In that case the denseness of the analytic wave function can be used as before (to separate the θ-ordered from the rest). This leads again to the "kinematical" crossing formula (29) which fortunately does not require any knowledge about how emulats depend on the interaction. From the crossing relation of formfactors one can derive the pair crossing of scattering amplitudes using LSZ reduction formulas.

The crossing of the elastic scattering amplitude was derived in a tour de force based on the use of the theory of multivariable analytic functions in [67]. The ordering limitation of the crossing identity which seems to have no counterpart in the formal LSZ derivation is in reality a property which exactly matches the non-overlapping limitations caused by hitting threshold singularities in the derivation of the LSZ reduction formalism from the rigorous Haag-Ruelle scattering theory [68].

Besides the nonperturbative new insights modular localization also led to ideas to use the mild short distance behavior of string-localized fields which allows couplings within the power-counting limit (the prerequisite for renormalizability) for *all spins.* In particular for s=1 string-localized potentials resolve the clash between localization and the Hilbert space structure of (m=0,s=1) representations by sacrificing the point-localization which is not only conceptually more reasonable than the BRST approach (which abandons the Hilbert space in favor of Krein spaces) but also permits to investigate problems which are not accessible to the BRST setting [38]. These problems are presently under

intense investigations [69].

The bridge between the Einstein-Jordan conundrum as the oldest Gedanken-experiment pointing towards consequences of modular localization and issues on the frontier of particle theory has been a source of great fascination to the author.

Note added: Thanks to John Stachel I became aware of an interesting and most comprehensive article on the Unruh effect by John Earman [71]. For a philosopher of science the interesting question is the concrete realization in terms of existing (or Gedanken-) observable hardware of Haag's quantum adaptation of the classical causal localization principle which is much more subtle than its intuitive support (finite spatial extension, finite duration of counter activation) which led him to present QFT in the LQP setting of local algebras. The present article on the other hand is concerned with the complete solution of the E-J conundrum in the concrete context of Jordan's chiral model (d=1+1 "photon") as an isomorphism between a restricted vacuum state (Jordan) with a global heat-bath system (Einstein). According to my best knowledge this model is the only known QFT which realises what has been called the "inverse Unruh effect" [24]. Concerning Hawking radiation, it is my firm conviction that the understanding of formation of black holes (which involves ideas which are not covered by modular localization [70]) is more important than finding a global state on the Kruskal extension which plays the same role for the localized region outside the Schwarzschild horizon as a restricted Minkowski vacuum. The modular thermalization is a property of the ensemble of all observables which are modular localized within the same region in the sense of Haag's LQP; but as in global statistical mechanics, the KMS condition is also satisfied by each individual operator of the localized ensemble. It is certainly not suitable as an egg-boiling device [71] since everything in such a causally closed world, including the "modular cook" will be boiled. Since a modular localization in $\mathcal{O}$ is also localized in every $\widehat{\mathcal{O}} \supset \mathcal{O}$, the $\mathcal{O}$-localized operators fulfill a continuous set of KMS relations with different modular Hamiltonians. This modular "tightness", which is totally absent in QM, shows that QFT is rightfully considered as a foundational QT; the prize to be paid for this enormous conceptual distance to QM is that all mathematically controlled approximation methods of the latter (notably single operator methods based on spectral resolution of selfadjoint operators) are powerless in QFT, and functional integral representations, which is shared between both, do not permit a mathematical control since the perturbative series diverges in QFT.

Of special interest to the author have been those manifestations of KMS properties of modular localization which do not require to think about a relation of the modular temperature to the one measured in terms of a thermometer as e.g. the (possibly logarithmically modified) *proportionality of the localization entropy to the dimensionless area* A/ε^2. It is rather improbable that Bekenstein's black hole entropy formula can be related to the vacuum polarization contribution near event horizons which are connected with the Hawking radiation (quantum matter in CST); more plausible is that it refers to the contribution of gravity degrees of freedom in a future QGR. Another problem is the

concept of information loss. Information theory has its conceptual home in QM where entanglement related to the decomposition of pure states with respect to a factorization into two subsystems leads to impure states in terms of averaging. The restriction of the QFT vacuum (more generally finite energy states) to a local subsystem obtained from modular localization however does *not need any averaging* over the causal complement. It is questionable whether information theory can be generalized to such situations.

The most rewarding results of wedge-localization is however the understanding of the formfactor *particle crossing* as a relic of the KMS cyclicity. The $\iota\pi$-strip analyticity needed for the return to physical formfactors together with the change from incoming ket particles to outgoing bra states corresponds to the KMS cyclicity together with the analyticity encoded in the "modular" temperature $T_{mod} = 1/2\pi$. This includes also the modular construction recipe for integrable models and the future promise to get a nonperturbative constructive hold on realistic models of QFT in a new top-to-bottom approach. The Unruh effect is certainly part of a bigger story about a structural property of QFT which is independent on such fleeting observer-dependent effects and doubts about relations of the modular temperature. In [73] it is shown that, although Unruh's correlations are thermal, the *local* temperature at a point fluctuates wildly around $T_{average} = 0$[28].

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